The Weekly Rigor

No. 1 “A mathematician is a machine for turning coffee into theorems.” June 28, 2014

Basic Logarithmic Properties
(Part 1)

INTRODUCTION

In the following, the italicized lower-case Roman letters a, b, c shall stand for any real positive numbers other than 1. They shall be used for bases. The letter m shall stand for any real number. It shall be used for exponents. The letters x and y shall stand for any positive real numbers. They shall be used as terms in the arguments of the logarithmic functions. Unless otherwise stated, all statements employing such variables shall be taken to hold universally, without exception.

Definition 1: $\log_a(x) = m$

if and only if

$a^m = x$.

In “$\log_a(x)$,” “a” is the “base” of the logarithm, and the expression between the parentheses--here denoted by “x”--is called the “argument” of the logarithmic function. The “m” in the first equation above is the “value” of the logarithmic function.

Remark: “A logarithm is an exponent.”

Example: $\log_2(8) = 3$,

since

$2^3 = 8$;

and vice versa.

Theorem 1 (Logarithmic Identity Property)**:**

$$\log_a(a) = 1.$$

Preliminary Remark: In words: The logarithm of any number to the base of the same number is always equal to 1.

Proof:

$$a^1 = a.$$

Therefore,

$$\log_a(a) = 1,$$

by Definition 1.

■

Theorem 2 (Logarithm of One)**:** $\log_a(1) = 0.$

Preliminary Remark: In words: The logarithm of 1 is always equal to 0.

Proof:

$$a^0 = 1.$$

Therefore,

$$\log_a(1) = 0,$$

by Definition 1.

■

Theorem 3 (Inverse Property of a Logarithm)**:**

$$\log_a(a^m) = m.$$

Proof:

$$a^m = a^m.$$

Therefore,

$$\log_a(a^m) = m,$$

by Definition 1.

■

Theorem 4 (Inverse Property of an Exponential)**:**

$$a^{\log_a(x)} = x.$$

Proof:

$$\log_a(x) = \log_a(x).$$

Therefore,

$$a^{\log_a(x)} = x,$$

by Definition 1.

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