# The Weekly Rigor

No. 7 "A mathematician is a machine for turning coffee into theorems." August 9, 2014


## Some Consequences of the Basic Logarithmic Properties

(Part 5)

**Theorem 20:** $-\log_a(x) = \log_a(1/x) = \log_{1/a}(x)$.

**Preliminary Remark:** "Base and argument are reciprocals."

**Proof:** $-\log_a(x) = -1 \cdot \log_a(x) = \log_a(x^{-1})$,

by Theorem 7 of *WR* no. 2.

$\log_a(x^{-1}) = \log_a(1/x)$.

Hence,

$-\log_a(x) = \log_a(1/x)$.

Furthermore,

$-\log_a(x) = \log_a(x)/-1 = \log_{a^{-1}}(x)$,

by Theorem 18.

$\log_{a^{-1}}(x) = \log_{1/a}(x)$.

So,

$-\log_a(x) = \log_{1/a}(x)$.

Therefore,

$-\log_a(x) = \log_a(1/x) = \log_{1/a}(x)$.

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**Theorem 21:** $\log_a(x) + \log_{1/a}(x) = 0$.

**Proof:** $\log_a(x) + \log_{1/a}(x) = \log_a(x) + \log_a(1/x)$,

by Theorem 20.

$\log_a(x) + \log_a(1/x) = \log_a(x/1) + \log_a(1/x)$.

Furthermore,

$\log_a(x/1) + \log_a(1/x) = 0$,

by Theorem 4. Therefore,

$\log_a(x) + \log_{1/a}(x) = 0$.

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**Theorem 22:** $x^{\log_a[\log_a(x)] \cdot [\log_a(x)]^{-1}} = \log_a(x)$.

**Proof:** $\log_a(x) = \log_a(x)$.

Hence,

$$\log_a(x) \cdot [\log_a(x)]^{-1} = 1.$$

So,

$$\{\log_a[\log_a(x)] \cdot [\log_a(x)]^{-1}\} \cdot \log_a(x) = \log_a[\log_a(x)].$$

Thus,

$$\log_a[x^{\log_a[\log_a(x)] \cdot [\log_a(x)]^{-1}}] = \log_a[\log_a(x)],$$

by Theorem 7 of *WR* no. 2. Therefore,

$$x^{\log_a[\log_a(x)] \cdot [\log_a(x)]^{-1}} = \log_a(x),$$

by Theorem 2.

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