The Weekly Rigor

No. 9 “A mathematician is a machine for turning coffee into theorems.” August 23, 2014

Counterexamples to Some Common Logarithmic Errors
(Part 2)

Counterexample 5: For some a, m, and x,

$$[\log_a(x)]^m \neq m \cdot \log_a(x).$$

Proof: Suppose by contradiction that

$$[\log_a(x)]^m = m \cdot \log_a(x).$$

Hence, with $a = m = x = 2$,

$$[\log_2(2)]^2 = 2 \cdot \log_2(2),$$

by substitution. But

$$\log_2(2) = 1,$$

by Theorem 1 of *WR* no. 1. So,

$$1^2 = 1 = 2 \cdot 1 = 2.$$

Contradiction.

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Counterexample 6: For some a, m, x, and y,

$$\log_a(x^m + y) \neq m \cdot \log_a(x + y).$$

Proof: Suppose by contradiction that

$$\log_a(x^m + y) = m \cdot \log_a(x + y).$$

Hence, with $a = x = 2$, $y = 1$, and $m = 0$,

$$\log_2(2^0 + 1) = \log_2(2) = 0 \cdot \log_2(2 + 1) = 0,$$

by substitution. But

$$\log_2(2) = 1,$$

by Theorem 1 of *WR* no. 1. So,

$$1 = 0.$$

Contradiction.

■

Counterexample 7: For some a, x, and y,

$$\frac{\log_a(x \bullet y)}{x} \neq \log_a(y).$$

Proof: Suppose by contradiction that

$$\frac{\log_a(x \bullet y)}{x} = \log_a(y).$$

Hence, with $a = y = 2$ and $x = ½$,

$$\frac{\log_2(½ \bullet 2)}{½} = \frac{\log_2(1)}{½} = \log_2(2),$$

by substitution. But

$$\log_2(1) = 0 \text{ and } \log_2(2) = 1,$$

by Theorems 2 and 1 of *WR* no. 1. So,

$$\frac{0}{½} = 0 = 1.$$

Contradiction.

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