The Weekly Rigor

No. 12 "A mathematician is a machine for turning coffee into theorems." September 13, 2014

Some Elementary Divisibility Properties
(Part 3)

Theorem 18: If $a \mid b$, then $a \mid -b$.

Proof: Suppose that $a \mid b$. Hence, $a \mid b\bullet(-1)$, by Theorem 10. Therefore,

$$a \mid -b.$$

■

Theorem 19: If $a \mid b$, then $-a \mid b$.

Proof: Suppose that $a \mid b$. Hence, $a \mid -b$, by Theorem 18. So, $ac = -b$ for some c, by Definition 1. Thus, $-ac = b$. Therefore,

$$-a \mid b,$$

by Definition 1.

■

Theorem 20: If $a \mid b$, then $-a \mid -b$.

Proof: Suppose that $a \mid b$. Hence, $a \mid -b$, by Theorem 18. Therefore,

$$-a \mid -b,$$

by Theorem 19.

■

Theorem 21: If both $a \mid b$ and $a \mid (b - c)$, then $a \mid c$.

Proof: Suppose that both $a \mid b$ and $a \mid (b - c)$. Hence, $a \mid (b + [-c])$. So, $a \mid -c$, by Theorem 17. Thus, $a \mid -[-c]$, by Theorem 18. Therefore,

$$a \mid c.$$

■

Theorem 22: $a \mid -a$.

Proof: $a \mid a$, by Theorem 3. Therefore,

$$a \mid -a,$$

by Theorem 18.

■

Theorem 23: $-1 \mid a$ for every a.

Proof: $1 \mid a$, by Theorem 4. Therefore,

$$-1 \mid a,$$

by Theorem 19.

■

Theorem 24: If both $a \mid b$ and $c \mid d$, then $ac \mid bd$.

Proof: Suppose that both $a \mid b$ and $c \mid d$. Hence, $ae = b$ and $cf = d$ for some e and f, by Definition 1. So, $aecf = ac(ef) = bd$, by substitution. Therefore,

$$ac \mid bd,$$

by Definition 1.

■

Theorem 25: If $ac \nmid bd$, then either $a \nmid b$ or $c \nmid d$.

Proof: By Theorem 24 and Definition 1.

■

Theorem 26: If both $a \mid b$ and $a \mid c$, then $a^2 \mid bc$.

Proof: Suppose that both $a \mid b$ and $a \mid c$. Hence, $aa \mid bc$, by Theorem 24. Therefore,

$$a^2 \mid bc.$$

■

Theorem 27: $a \mid b$ if and only if $ac \mid bc$ for $c \neq 0$.

Proof: Suppose that $a \mid b$. $c \mid c$, by Theorem 3. Therefore, $ac \mid bc$ for $c \neq 0$, by Theorem 24.

Suppose that $ac \mid bc$ for $c \neq 0$. Hence, $acd = bc$ for some d, by Definition 1. So, since $c \neq 0$, $ad = b$. Therefore, $a \mid b$, by Definition 1.

■