The Weekly Rigor

No. 17 "A mathematician is a machine for turning coffee into theorems." October 18, 2014

Even and Odd Integers: Some Consequences of the Basic Properties
(Part 2)

Extending a theme from *WR* no. 16, we shall apply the table presented in *WR* no. 15 to analyze two-term expressions each term of which consists of two integer factors, viz.,

$$ax + by.$$

As was proved in the previous issue, we will also be covering all the positive integer powers of the factors at the same time.

Employing the basic table

m	*n*	*m* + *n*	*mn*
EVEN	EVEN	EVEN	EVEN
EVEN	ODD	ODD	EVEN
ODD	EVEN	ODD	EVEN
ODD	ODD	EVEN	ODD

and listing all the possible combinations of even ("E") and odd ("O") for the factors of *ax* +*by*, we get the following table:

ROW#	*a*	*x*	*b*	*y*	*ax*	*by*	*ax* + *by*
1	E	E	E	E	E	E	**E**
2	E	E	E	O	E	E	**E**
3	E	E	O	E	E	E	E
4	E	E	O	O	E	O	**O**
5	E	O	E	E	E	E	E
6	E	O	E	O	E	E	**E**
7	E	O	O	E	E	E	E
8	E	O	O	O	E	O	**O**
9	O	E	E	E	E	E	E
10	O	E	E	O	E	E	E
11	O	E	O	E	E	E	E
12	O	E	O	O	E	O	O
13	O	O	E	E	O	E	O
14	O	O	E	O	O	E	O
15	O	O	O	E	O	E	O
16	O	O	O	O	O	O	**E**

I have bolded whether the final sum is even or odd for those combinations of factors that are distinctive, ignoring the order of two factors and/or two terms. (Cf. rows 1, 2, 4, 6, 8, and 16.)

Some applications of the new table: If both *a* and *b* are even, then $ax + by$ is even. If both *a* and *b* are odd and $ax + by$ is even, then either both *x* and *y* are even or both *x* and *y* are odd. If both *a* and *b* are odd and $ax + b$ is even, then *x* is odd. (Let $y = 1$.) If *x* is even and *a*, *b*, and *y* are odd, then $ax + by$ is odd.

Picking out the distinctive rows from the last table, we have

OLD ROW#	*a*	*x*	*b*	*y*	*ax*	*by*	*ax* + *by*
1	E	E	E	E	E	E	**E**
2	E	E	E	O	E	E	**E**
4	E	E	O	O	E	O	**O**
6	E	O	E	O	E	E	**E**
8	E	O	O	O	E	O	**O**
16	O	O	O	O	O	O	**E**

We can use this brief table to consider three-term expressions of the form

$$ax + by + cz$$

Both factors of the term *cz* are even, odd, or just one factor is even. Hence, there are only three possibilities regarding *cz* we need to consider in constructing the next table.

ROW#	*a*	*x*	*b*	*y*	*c*	*z*	*ax*	*by*	*cz*	*ax* + *by*	(*ax* + *by*) + *cz*
1	E	E	E	E	E	E	E	E	E	E	E
2	E	E	E	E	E	O	E	E	E	E	E
3	E	E	E	E	O	O	E	E	O	E	O
4	E	E	E	O	E	E	E	E	E	E	E
5	E	E	E	O	E	O	E	E	E	E	E
6	E	E	E	O	O	O	E	E	O	E	O
7	E	E	O	O	E	E	E	O	E	O	O
8	E	E	O	O	E	O	E	O	E	O	O
9	E	E	O	O	O	O	E	O	O	O	E
10	E	O	E	O	E	E	E	E	E	E	E
11	E	O	E	O	E	O	E	E	E	E	E
12	E	O	E	O	O	O	E	E	O	E	O
13	E	O	O	O	E	E	E	O	E	O	O
14	E	O	O	O	E	O	E	O	E	O	O
15	E	O	O	O	O	O	E	O	O	O	E
16	O	O	O	O	E	E	O	O	E	E	E
17	O	O	O	O	E	O	O	O	E	E	E
18	O	O	O	O	O	O	O	O	O	E	O