The Weekly Rigor

No. 20 "A mathematician is a machine for turning coffee into theorems." November 8, 2014

51 Problems in Calculating Limits Using L'Hôpital's Rule with Solutions

(Part 1)

INTRODUCTION

Problems 1-23 involve the indeterminate forms $\frac{\infty}{\infty}$ and $\frac{0}{0}$. Problems 24-31 involve indeterminate products, $0 \cdot \infty$. Problems 32-40 involve indeterminate differences, $\infty - \infty$. Problems 41-51 involve indeterminate powers, 0^0, ∞^0, and 1^∞.

In the solutions, the applications of L'Hôpital's Rule is denoted by "$\overset{LH}{=}$."

The appendix, which begins at *WR* no. 28, includes the problems stated in a random order, to help the student test his skill and detect which kinds of problems need special attention. The answers immediately following the problems in random order include the number of the original problem. Furthermore, the appendix includes a table of mathematical facts useful in calculating limits using L'Hôpital's Rule. Lastly, two applications of the rule are presented.

PROBLEMS

Find the following limits.

1. $\lim_{x\to\infty} \frac{e^{3x}}{x^2}$
2. $\lim_{x\to 0} \frac{\tan(2x)}{\ln(1+x)}$
3. $\lim_{x\to 0} \frac{\sin(x)-x}{x^3}$
4. $\lim_{x\to 0} \frac{e^x-1}{\sin(2x)}$
5. $\lim_{x\to 1} \frac{1-x+\ln(x)}{1+\cos(\pi x)}$
6. $\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta}$
7. $\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\theta}$
18. $\lim_{x\to 0} \frac{10^x-2^x}{6x}$
19. $\lim_{x\to 0} \frac{x^2}{\ln(\cos(x))}$
20. $\lim_{x\to 0} \frac{x\cdot 2^x}{2^x-1}$
21. $\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln(x)}$
22. $\lim_{x\to\infty} \frac{e^x+x^2}{e^x-x}$
23. $\lim_{x\to 0} \frac{2\cos(\theta)-2}{e^\theta-\theta-1}$
24. $\lim_{\theta\to\infty} x^2e^{-x}$
35. $\lim_{x\to 1} \left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right)$
36. $\lim_{x\to 0} \left(\frac{1}{x}-\frac{1}{\sin(x)}\right)$
37. $\lim_{x\to\infty} \left(\sqrt{x^2+3x}-x\right)$
38. $\lim_{x\to 1^+} \left(\frac{1}{x-1}-\frac{1}{\ln(x)}\right)$
39. $\lim_{x\to\infty} \left(\frac{x^2}{x-1}-\frac{x^2}{x+1}\right)$
40. $\lim_{x\to\infty} \left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$
41. $\lim_{x\to 0^+} (1+\sin(4x))^{\cot(x)}$

8. $\lim_{t\to 0}\frac{1-\cos(t)}{t^2}$

9. $\lim_{t\to 0^+}\frac{\ln(t)}{\csc(t)}$

10. $\lim_{x\to 0}\frac{\cos(x)+2x-1}{3x}$

11. $\lim_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos(2x)}$

12. $\lim_{x\to \frac{\pi}{2}^-}\frac{4\tan(x)}{1+\sec(x)}$

13. $\lim_{x\to\infty}\frac{\ln(x)}{\sqrt{x}}$

14. $\lim_{x\to\infty}\frac{\ln(x)}{\ln(x+1)}$

15. $\lim_{x\to 0}\frac{1-\cos(x)}{\sin(x)-x+x^2}$

16. $\lim_{x\to 0}\frac{\sin(x)-2x}{x}$

17. $\lim_{x\to\infty}\frac{\ln(x)}{x^{10}}$

25. $\lim_{x\to 0^+} x\ln(x)$

26. $\lim_{x\to 0^+} x^2\ln(x)$

27. $\lim_{x\to \frac{\pi}{2}}(2x-\pi)\sec(x)$

28. $\lim_{x\to \frac{\pi}{2}}\tan(x)\ln(\sin(x))$

29. $\lim_{x\to \frac{\pi}{2}^-}\left(x-\frac{\pi}{2}\right)\sec(x)$

30. $\lim_{x\to 0^+}\sin(x)\ln(\sin(x))$

31. $\lim_{x\to 0} x\cot(x)$

32. $\lim_{x\to\infty}[\ln(2x)-\ln(x+1)]$

33. $\lim_{x\to \frac{\pi}{2}^-}(\sec(x)-\tan(x))$

34. $\lim_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$

42. $\lim_{x\to 0^+} x^x$

43. $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$

44. $\lim_{x\to 1^+} x^{\frac{1}{1-x}}$

45. $\lim_{x\to\infty}[\ln(x)]^{\frac{1}{x}}$

46. $\lim_{x\to 0^+} x^{\frac{-1}{\ln(x)}}$

47. $\lim_{x\to\infty}(1+2x)^{\frac{1}{2\ln(x)}}$

48. $\lim_{x\to 0}(e^x+x)^{\frac{2}{x}}$

49. $\lim_{h\to 0}(1+hx)^{\frac{1}{h}}$

50. $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$

51. $\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{nt}$