The Weekly Rigor

No. 21 "A mathematician is a machine for turning coffee into theorems." November 15, 2014

51 Problems in Calculating Limits Using L'Hôpital's Rule with Solutions
(Part 2)

ANSWERS

1. ∞
2. 2
3. $\frac{-1}{6}$
4. $\frac{1}{2}$
5. $\frac{-1}{\pi^2}$
6. 1
7. 0
8. $\frac{1}{2}$
9. 0
10. $\frac{2}{3}$
11. $\frac{1}{2}$
12. 4
13. 0
14. 1
15. $\frac{1}{2}$
16. -1
17. 0
18. $\frac{\ln(5)}{6}$
19. -2
20. $\frac{1}{\ln(2)}$
21. 1
22. 1
23. -2
24. 0
25. 0
26. 0
27. -2
28. 0
29. 1
30. 0
31. 1
32. $\ln(2)$
33. 0
34. $\frac{-1}{2}$
35. $\frac{1}{2}$
36. 0
37. $\frac{3}{2}$
38. $\frac{-1}{2}$
39. 2
40. 1
41. e^4
42. 1
43. e
44. $\frac{1}{e}$
45. 1
46. $\frac{1}{e}$
47. $e^{\frac{1}{2}}$
48. e^4
49. e^x
50. e^x
51. e^{rt}

SOLUTIONS

1. $$\lim_{x\to\infty}\frac{e^{3x}}{x^2}\overset{LH}{\cong}\lim_{x\to\infty}\frac{3e^{3x}}{2x}\overset{LH}{\cong}\lim_{x\to\infty}\frac{9e^{3x}}{2}=\infty.$$

2. $$\lim_{x\to 0}\frac{\tan(2x)}{\ln(1+x)}\overset{LH}{\cong}\lim_{x\to 0}\frac{2\sec^2(2x)}{\frac{1}{1+x}}=\lim_{x\to 0}\frac{2\sec^2(2x)}{1}\cdot\frac{1+x}{1}=\frac{2\sec^2(0)}{1}\cdot\frac{1+0}{1}=2\cdot 1=2.$$

3. $$\lim_{x\to 0}\frac{\sin(x)-x}{x^3}\overset{LH}{\cong}\lim_{x\to 0}\frac{\cos(x)-1}{3x^2}\overset{LH}{\cong}\lim_{x\to 0}\frac{-\sin(x)}{6x}\overset{LH}{\cong}\lim_{x\to 0}\frac{-\cos(x)}{6}=\frac{-\cos(0)}{6}=-\frac{1}{6}.$$

4. $$\lim_{x\to 0}\frac{e^x-1}{\sin(2x)}\overset{LH}{\cong}\lim_{x\to 0}\frac{e^x}{2\cos(2x)}=\frac{e^0}{2\cos(0)}=\frac{1}{2(1)}=\frac{1}{2}.$$

5. $$\lim_{x\to 1}\frac{1-x+\ln(x)}{1+\cos(\pi x)}\overset{LH}{\cong}\lim_{x\to 1}\frac{-1+\frac{1}{x}}{-\pi\sin(\pi x)}=\lim_{x\to 1}\frac{x-1}{\pi x\sin(\pi x)}\overset{LH}{\cong}\lim_{x\to 1}\frac{1}{\pi\sin(\pi x)+\pi^2 x\cos(\pi x)}=$$
$$=\frac{1}{\pi\sin(\pi)+\pi^2\cos(\pi)}=\frac{1}{\pi(0)+\pi^2(-1)}=\frac{-1}{\pi^2}.$$

6. $$\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}\overset{LH}{\cong}\lim_{\theta\to 0}\frac{\cos(\theta)}{1}=\frac{\cos(0)}{1}=\frac{1}{1}=1.$$

7. $$\lim_{\theta\to 0}\frac{1-\cos(\theta)}{\theta}\overset{LH}{\cong}\lim_{\theta\to 0}\frac{\sin(\theta)}{1}=\frac{\sin(0)}{1}=\frac{0}{1}=0.$$

8. $$\lim_{t\to 0}\frac{1-\cos(t)}{t^2}\overset{LH}{\cong}\lim_{t\to 0}\frac{\sin(t)}{2t}\overset{LH}{\cong}\lim_{t\to 0}\frac{\cos(t)}{2}=\frac{\cos(0)}{2}=\frac{1}{2}.$$