The Weekly Rigor

No. 23 “A mathematician is a machine for turning coffee into theorems.” November 29, 2014

51 Problems in Calculating Limits Using L’Hôpital’s Rule with Solutions

(Part 4)

23. $$\lim_{\theta\to 0}\frac{2\cos(\theta)-2}{e^{\theta}-\theta-1}\overset{LH}{=}\lim_{\theta\to 0}\frac{-2\sin(\theta)}{e^{\theta}-1}\overset{LH}{=}\lim_{\theta\to 0}\frac{-2\cos(\theta)}{e^{\theta}}=\frac{-2\cos(0)}{e^{0}}=\frac{-2(1)}{1}=\frac{-2}{1}=-2.$$

24. $$\lim_{x\to\infty}x^2e^{-x}=\lim_{x\to\infty}\frac{x^2}{e^x}\overset{LH}{=}\lim_{x\to\infty}\frac{2x}{e^x}\overset{LH}{=}\lim_{x\to\infty}\frac{2}{e^x}=0.$$

25. $$\lim_{x\to 0^+}x\ln(x)=\lim_{x\to 0^+}\frac{\ln(x)}{\left(\frac{1}{x}\right)}=\lim_{x\to 0^+}\frac{\ln(x)}{x^{-1}}\overset{LH}{=}\lim_{x\to 0^+}\frac{\frac{1}{x}}{-x^{-2}}=\lim_{x\to 0^+}\frac{\left(\frac{1}{x}\right)}{\left(\frac{-1}{x^2}\right)}=\lim_{x\to 0^+}\frac{1}{x}\cdot\frac{x^2}{-1}=$$
$$=-\lim_{x\to 0^+}x=-0=0.$$

26. $$\lim_{x\to 0^+}x^2\ln(x)=\lim_{x\to 0^+}\frac{\ln(x)}{\left(\frac{1}{x^2}\right)}=\lim_{x\to 0^+}\frac{\ln(x)}{x^{-2}}\overset{LH}{=}\lim_{x\to 0^+}\frac{\frac{1}{x}}{-2x^{-3}}=\lim_{x\to 0^+}\frac{\left(\frac{1}{x}\right)}{\left(\frac{-2}{x^3}\right)}=\lim_{x\to 0^+}\frac{1}{x}\cdot\frac{x^3}{-2}=$$
$$=\frac{-1}{2}\lim_{x\to 0^+}x^2=\frac{-1}{2}(0)^2=0.$$

27. $$\lim_{x\to\frac{\pi}{2}}(2x-\pi)\sec(x)=\lim_{x\to\frac{\pi}{2}}\frac{2x-\pi}{\cos(x)}\overset{LH}{=}\lim_{x\to\frac{\pi}{2}}\frac{2}{-\sin(x)}=\frac{2}{-\sin\left(\frac{\pi}{2}\right)}=\frac{2}{-1}=-2.$$

28. $$\lim_{x\to\frac{\pi}{2}}\tan(x)\ln(\sin(x))=\lim_{x\to\frac{\pi}{2}}\frac{\ln(\sin(x))}{\cot(x)}\overset{LH}{=}\lim_{x\to\frac{\pi}{2}}\frac{\left(\frac{\cos(x)}{\sin(x)}\right)}{-\csc^2(x)}=\lim_{x\to\frac{\pi}{2}}\frac{\cos(x)}{\sin(x)}\cdot\frac{-1}{\csc^2(x)}=$$
$$=\lim_{x\to\frac{\pi}{2}}\frac{\cos(x)}{\sin(x)}\cdot\frac{-\sin^2(x)}{1}=-\lim_{x\to\frac{\pi}{2}}\cos(x)\sin(x)=-\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)=-(0)(1)=0.$$

29. $\lim\limits_{x\to\frac{\pi}{2}^-}\left(x-\frac{\pi}{2}\right)\sec(x) = \lim\limits_{x\to\frac{\pi}{2}^-}\frac{x-\frac{\pi}{2}}{\cos(x)} \overset{LH}{\cong} \lim\limits_{x\to\frac{\pi}{2}^-}\frac{1}{-\sin(x)} = \frac{1}{-\sin\left(\frac{\pi}{2}\right)} = \frac{1}{-1} = 1.$

30. $\lim\limits_{x\to 0^+}\sin(x)\ln(\sin(x)) = \lim\limits_{x\to 0^+}\frac{\ln(\sin(x))}{\frac{1}{\sin(x)}} \overset{LH}{\cong} \lim\limits_{x\to 0^+}\frac{\left(\frac{\cos(x)}{\sin(x)}\right)}{\left(\frac{-\cos(x)}{\sin^2(x)}\right)} = -\lim\limits_{x\to 0^+}\frac{\cos(x)}{\sin(x)}\cdot\frac{\sin^2(x)}{\cos(x)} =$
$= -\lim\limits_{x\to 0^+}\sin(x) = -\sin(0) = 0.$

31. $\lim\limits_{x\to 0}x\cot(x) = \lim\limits_{x\to 0}\frac{x}{\tan(x)} \overset{LH}{\cong} \lim\limits_{x\to 0}\frac{1}{\sec^2(x)} = \lim\limits_{x\to 0}\cos^2(x) = \cos^2(0) = 1.$

32. $\lim\limits_{x\to\infty}[\ln(2x)-\ln(x+1)] = \lim\limits_{x\to\infty}\ln\left(\frac{2x}{x+1}\right) = \ln\left(\lim\limits_{x\to\infty}\frac{2x}{x+1}\right) \overset{LH}{\cong} \ln\left(\lim\limits_{x\to\infty}\frac{2}{1}\right) = \ln\left(\lim\limits_{x\to\infty}2\right) =$
$= \ln(2).$

33. $\lim\limits_{x\to\frac{\pi}{2}^-}(\sec(x)-\tan(x)) = \lim\limits_{x\to\frac{\pi}{2}^-}\left(\frac{1}{\cos(x)}-\frac{\sin(x)}{\cos(x)}\right) = \lim\limits_{x\to\frac{\pi}{2}^-}\frac{1-\sin(x)}{\cos(x)} \overset{LH}{\cong} \lim\limits_{x\to\frac{\pi}{2}^-}\frac{-\cos(x)}{-\sin(x)} =$
$= \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0.$

34. $\lim\limits_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right) = \lim\limits_{x\to 0}\frac{x-e^x+1}{xe^x-x} \overset{LH}{\cong} \lim\limits_{x\to 0}\frac{1-e^x}{xe^x+e^x-1} \overset{LH}{\cong} \lim\limits_{x\to 0}\frac{-e^x}{xe^x+2e^x} = \frac{-e^0}{(0)e^0+2e^0} =$
$= \frac{-1}{0+2} = \frac{-1}{2}.$

35. $\lim\limits_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right) = \lim\limits_{x\to 1}\frac{x\ln(x)-(x+1)}{(x-1)\ln(x)} = \lim\limits_{x\to 1}\frac{x\ln(x)-x-1}{x\ln(x)-\ln(x)} \overset{LH}{\cong} \lim\limits_{x\to 1}\frac{\ln(x)+1-1}{\ln(x)+1-\frac{1}{x}} =$
$= \lim\limits_{x\to 1}\frac{\ln(x)}{\left(\frac{x\ln(x)+x-1}{x}\right)} = \lim\limits_{x\to 1}\frac{x\ln(x)}{x\ln(x)+x-1} \overset{LH}{\cong} \lim\limits_{x\to 1}\frac{\ln(x)+1}{\ln(x)+1+1} = \lim\limits_{x\to 1}\frac{\ln(x)+1}{\ln(x)+2} =$
$= \frac{\ln(1)+1}{\ln(1)+2} = \frac{0+1}{0+2} = \frac{1}{2}.$