The Weekly Rigor

No. 28 "A mathematician is a machine for turning coffee into theorems." January 3, 2015

51 Problems in Calculating Limits Using L'Hôpital's Rule with Solutions
(Part 9)

APPENDIX

PROBLEMS IN RANDOM ORDER

Find the following limits.

1. $\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln(x)}$
2. $\lim_{x\to 0} \frac{x\cdot 2^x}{2^x-1}$
3. $\lim_{x\to 0} \frac{10^x-2^x}{6x}$
4. $\lim_{x\to \infty} \frac{e^x+x^2}{e^x-x}$
5. $\lim_{x\to 0} \frac{e^x+e^{-x}-2}{1-\cos(2x)}$
6. $\lim_{x\to 0} \frac{x^2}{\ln(\cos(x))}$
7. $\lim_{x\to \infty} \frac{\ln(x)}{\sqrt{x}}$
8. $\lim_{x\to \infty} \frac{\ln(x)}{\ln(x+1)}$
9. $\lim_{x\to 0^+} x\ln(x)$
10. $\lim_{x\to 0} \left(\frac{1}{e^x-1}-\frac{1}{x}\right)$
11. $\lim_{x\to 0} \left(\frac{1}{x}-\frac{1}{\sin(x)}\right)$
12. $\lim_{n\to \infty} \left(1+\frac{r}{n}\right)^{nt}$
13. $\lim_{x\to 0} (1+x)^{\frac{1}{x}}$
14. $\lim_{x\to \frac{\pi}{2}} \tan(x)\ln(\sin(x))$
15. $\lim_{x\to \infty} \left(\sqrt{x^2+3x}-x\right)$
16. $\lim_{x\to \frac{\pi}{2}} (2x-\pi)\sec(x)$
17. $\lim_{x\to 0^+} (1+\sin(4x))^{\cot(x)}$
18. $\lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)-x+x^2}$
19. $\lim_{\theta\to \infty} x^2 e^{-x}$
20. $\lim_{x\to 1} \frac{1-x+\ln(x)}{1+\cos(\pi x)}$
21. $\lim_{x\to \frac{\pi}{2}^-} (\sec(x)-\tan(x))$
22. $\lim_{x\to \infty} \frac{e^{3x}}{x^2}$
23. $\lim_{x\to \infty} \left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$
24. $\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta}$
25. $\lim_{x\to 0} \frac{\sin(x)-x}{x^3}$
26. $\lim_{x\to 0} x\cot(x)$
27. $\lim_{x\to \infty} (1+2x)^{\frac{1}{2\ln(x)}}$
28. $\lim_{x\to 0^+} x^2\ln(x)$
29. $\lim_{x\to 0^+} x^x$
30. $\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\theta}$
31. $\lim_{x\to 0^+} x^{\frac{-1}{\ln(x)}}$
32. $\lim_{n\to \infty} \left(1+\frac{x}{n}\right)^n$
33. $\lim_{x\to \infty} \left(\frac{x^2}{x-1}-\frac{x^2}{x+1}\right)$
34. $\lim_{x\to 0} (e^x+x)^{\frac{2}{x}}$
35. $\lim_{x\to 0} \frac{2\cos(\theta)-2}{e^\theta-\theta-1}$
36. $\lim_{t\to 0} \frac{1-\cos(t)}{t^2}$
37. $\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta}$
38. $\lim_{x\to 0} \frac{\tan(2x)}{\ln(1+x)}$
39. $\lim_{x\to 1^+} x^{\frac{1}{1-x}}$
40. $\lim_{x\to 1^+} \left(\frac{1}{x-1}-\frac{1}{\ln(x)}\right)$
41. $\lim_{h\to 0} (1+hx)^{\frac{1}{h}}$
42. $\lim_{x\to \infty} [\ln(x)]^{\frac{1}{x}}$
43. $\lim_{x\to \infty} \frac{\ln(x)}{x^{10}}$
44. $\lim_{x\to \frac{\pi}{2}^-} \frac{4\tan(x)}{1+\sec(x)}$
45. $\lim_{x\to 0} \frac{\cos(x)+2x-1}{3x}$
46. $\lim_{x\to 0} \frac{\sin(x)-2x}{x}$
47. $\lim_{t\to 0^+} \frac{\ln(t)}{\csc(t)}$
48. $\lim_{x\to 0} \frac{e^x-1}{\sin(2x)}$
49. $\lim_{x\to 0^+} \sin(x)\ln(\sin(x))$
50. $\lim_{x\to 1} \left(\frac{x}{x-1}-\frac{1}{\ln(x)}\right)$
51. $\lim_{x\to \infty} [\ln(2x)-\ln(x+1)]$

ANSWERS

1. 1 (#21)
2. $\frac{1}{\ln(2)}$ (#20)
3. $\frac{\ln(5)}{6}$ (#18)
4. 1 (#22)
5. $\frac{1}{2}$ (#11)
6. −2 (#19)
7. 0 (#13)
8. 1 (#14)
9. 0 (#25)
10. $\frac{-1}{2}$ (#34)
11. 0 (#36)
12. e^{rt} (#51)
13. e (#43)
14. 0 (#28)
15. $\frac{3}{2}$ (#37)
16. −2 (#27)
17. e^4 (#41)
18. $\frac{1}{2}$ (#15)
19. 0 (#24)
20. $\frac{-1}{\pi^2}$ (#5)
21. 0 (#33)
22. ∞ (#1)
23. 1 (#40)
24. 1 (#6)
25. $\frac{-1}{6}$ (#3)
26. 1 (#31)
27. $e^{\frac{1}{2}}$ (#47)
28. 0 (#26)
29. 1 (#42)
30. 0 (#7)
31. $\frac{1}{e}$ (#46)
32. e^x (#50)
33. 2 (#39)
34. e^4 (#48)
35. −2 (#23)
36. $\frac{1}{2}$ (#8)
37. 1 (#6)
38. 2 (#2)
39. $\frac{1}{e}$ (#44)
40. $\frac{-1}{2}$ (#38)
41. e^x (#49)
42. 1 (#45)
43. 0 (#17)
44. 4 (#12)
45. $\frac{2}{3}$ (#10)
46. −1 (#16)
47. 0 (#9)
48. $\frac{1}{2}$ (#4)
49. 0 (#30)
50. $\frac{1}{2}$ (#35)
51. ln(2) (#32)