

The Weekly Rigor

No. 31

“A mathematician is a machine for turning coffee into theorems.”

January 24, 2015

101 Problems in Calculating Trigonometric Limits with Solutions (Part 2)

PROBLEMS

Find the following limits.

1. $\lim_{\theta \rightarrow 0} \frac{\theta}{\sin(\theta)}$

2. $\lim_{\theta \rightarrow 0} \frac{3\sin(\theta)}{\theta}$

3. $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{\theta}$

4. $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{\sin(4\theta)}$

5. $\lim_{\theta \rightarrow 0^+} \frac{\theta}{\sin(\sqrt{\theta})}$

6. $\lim_{\theta \rightarrow 0^+} \frac{\sin(\theta)}{\sqrt{\theta}}$

7. $\lim_{\theta \rightarrow 0} \theta \csc(\theta)$

8. $\lim_{\theta \rightarrow 0} \frac{\sin(-3\theta)}{4\theta}$

9. $\lim_{\theta \rightarrow 0} \frac{\sin(a\theta)}{b\theta}$

10. $\lim_{\theta \rightarrow 0} \frac{\sin(a\theta)}{\sin(b\theta)}$

11. $\lim_{\theta \rightarrow 0} \frac{\sin^2(k\theta)}{\theta^2}$

12. $\lim_{\theta \rightarrow 0} \sec(\theta)$

13. $\lim_{\theta \rightarrow 0} 3 \cos(\theta)$

14. $\lim_{\theta \rightarrow 0} \frac{1}{\cos(\theta)}$

35. $\lim_{\theta \rightarrow 0} \frac{\tan(3\theta)}{\tan(4\theta)}$

36. $\lim_{\theta \rightarrow 0} \frac{\tan(a\theta)}{\tan(b\theta)}$

37. $\lim_{\theta \rightarrow 0} \frac{\theta}{\cos(\frac{\pi}{2}-\theta)}$

38. $\lim_{\theta \rightarrow 0} \frac{\cos(\theta+\frac{\pi}{2})}{\theta}$

39. $\lim_{\theta \rightarrow 0} \frac{\cos(\frac{\pi}{2}-\theta)}{\theta}$

40. $\lim_{\theta \rightarrow 0} \frac{\theta}{\sin(\theta+\frac{\pi}{2})}$

41. $\lim_{\theta \rightarrow 0} \frac{\sin(\frac{\pi}{2}-\theta) \tan(\pi+\theta)}{\theta}$

42. $\lim_{\theta \rightarrow 0} \sin\left(\frac{\pi}{2} + \theta\right) \cos\left(\frac{\pi}{2} + \theta\right)$

43. $\lim_{\theta \rightarrow 0} \frac{\cos(\frac{\pi}{2}-a\theta)}{b\theta}$

44. $\lim_{\theta \rightarrow 0} \frac{\cos(\frac{\pi}{2}-a\theta)}{\cos(\frac{\pi}{2}-b\theta)}$

45. $\lim_{\theta \rightarrow 0} \frac{\sin(\frac{\pi}{2}-a\theta)}{\sin(\frac{\pi}{2}-b\theta)}$

46. $\lim_{\theta \rightarrow 0} [\sin(\pi - \theta) + \tan(\pi - \theta)]$

47. $\lim_{\theta \rightarrow 0} \frac{\tan(a\theta)}{\sin(b\theta)}$

48. $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{1-\cos(\theta)}$

69. $\lim_{\theta \rightarrow 0} \frac{\theta^2}{1-\cos^2(2\theta)}$

70. $\lim_{\theta \rightarrow 0} \frac{\sec(6\theta) \tan(3\theta)}{\theta}$

71. $\lim_{\theta \rightarrow 0} \theta^2 \cot^2(4\theta)$

72. $\lim_{\theta \rightarrow 0} \frac{\tan(\pi-\theta)-\theta}{\sin(\theta+\pi)}$

73. $\lim_{\theta \rightarrow 0} \frac{\cos(a\theta) \tan(a\theta)}{b\theta}$

74. $\lim_{\theta \rightarrow 0} \frac{\cos(a\theta) \tan(a\theta)}{\cos(b\theta) \tan(b\theta)}$

75. $\lim_{\theta \rightarrow 0} \frac{\sin^2(\theta)+2 \cos(\theta)-2}{\cos^2(\theta)-\sin(\theta)-1}$

76. $\lim_{\theta \rightarrow 0} \frac{\sin(2\theta)-\tan(2\theta)}{\theta^2}$

77. $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)-2\theta}{\theta}$

78. $\lim_{\theta \rightarrow 0} \frac{3-\csc(\theta)}{7-\cot(\theta)}$

79. $\lim_{\theta \rightarrow 0} \frac{\theta \cos(\theta)-\sin(\theta)}{\theta}$

80. $\lim_{\theta \rightarrow 0} \frac{\sin(2\theta) \tan(\theta)}{3\theta}$

81. $\lim_{\theta \rightarrow 0} \frac{\sin(2\theta)+\tan(\theta)}{3\theta}$

82. $\lim_{\theta \rightarrow 0} \frac{\tan(\theta)-\sin(\theta)}{\theta^2}$

15. $\lim_{\theta \rightarrow 0} \frac{\cos(\theta)}{\theta}$
16. $\lim_{\theta \rightarrow 0} \frac{\theta}{\cos(\theta)}$
17. $\lim_{\theta \rightarrow 0} \frac{\cos(3\theta)}{\cos(4\theta)}$
18. $\lim_{\theta \rightarrow 0} \frac{\cos(a\theta)}{\cos(b\theta)}$
19. $\lim_{\theta \rightarrow 0} \frac{1-\cos(\theta)}{\theta}$
20. $\lim_{\theta \rightarrow 0} \frac{\cos(\theta)-1}{\theta}$
21. $\lim_{\theta \rightarrow 0} \frac{1-\cos(\theta)}{\theta^2}$
22. $\lim_{\theta \rightarrow 0} \frac{2\cos(\theta)-2}{3\theta}$
23. $\lim_{\theta \rightarrow 0} \frac{\theta^2}{1-\cos(\theta)}$
24. $\lim_{\theta \rightarrow 0} \frac{1-\cos(5\theta)}{\cos(7\theta)-1}$
25. $\lim_{\theta \rightarrow 0} \frac{1-2\theta^2-2\cos(\theta)+\cos^2(\theta)}{\theta^2}$
26. $\lim_{\theta \rightarrow 0} \frac{1-\cos(\theta)}{\theta^2}$
27. $\lim_{\theta \rightarrow 0} \frac{1-\cos(3\theta)}{\theta^2}$
28. $\lim_{\theta \rightarrow 0} \frac{1-\cos(a\theta)}{b\theta}$
29. $\lim_{\theta \rightarrow 0} \tan(\theta)$
30. $\lim_{\theta \rightarrow 0} 3\tan(\theta)$
31. $\lim_{\theta \rightarrow 0} \frac{\tan(\theta)}{\theta}$
32. $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan(\theta)}$
33. $\lim_{\theta \rightarrow 0} \frac{\tan(3\theta)}{\theta}$
34. $\lim_{\theta \rightarrow 0} \theta \cot(\theta)$
49. $\lim_{\theta \rightarrow 0} \frac{5\theta+\sin(3\theta)}{\tan(4\theta)-7\theta\cos(6\theta)}$
50. $\lim_{\theta \rightarrow 0} \frac{1-\cos^3(\theta)}{\sin^2(\theta)}$
51. $\lim_{\theta \rightarrow 0} \frac{2+\sin(\theta)}{3+\theta}$
52. $\lim_{\theta \rightarrow 0} \frac{\theta^2+1}{\theta+\cos(\theta)}$
53. $\lim_{\theta \rightarrow 0} \frac{\theta+\tan(\theta)}{\sin(\theta)}$
54. $\lim_{\theta \rightarrow 0} \frac{1-\cos(3\theta)}{\theta \sin(\theta)}$
55. $\lim_{\theta \rightarrow 0} \frac{1-\cos(2\theta)}{\theta \sin(\theta)}$
56. $\lim_{\theta \rightarrow 0} \frac{1-\cos(\theta)}{\theta \sin(\theta)}$
57. $\lim_{\theta \rightarrow 0} \frac{\cos(\theta)}{\theta \csc(\theta)}$
58. $\lim_{\theta \rightarrow 0} \sin(2\theta) \cot(\theta)$
59. $\lim_{\theta \rightarrow 0} \frac{1-\cos(\theta)}{\sin(\theta)}$
60. $\lim_{\theta \rightarrow 0} \frac{1-\cos(\theta)}{\tan(\theta)}$
61. $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\tan(\theta)}$
62. $\lim_{\theta \rightarrow 0} \frac{2\sin(\theta)-\sin(2\theta)}{\theta \cos(\theta)}$
63. $\lim_{\theta \rightarrow 0} \frac{\tan(\theta)-\sin(\theta)}{\theta \cos(\theta)}$
64. $\lim_{\theta \rightarrow 0} \frac{\csc(\theta)-\cot(\theta)}{\sin(\theta)}$
65. $\lim_{\theta \rightarrow 0} \frac{2\theta+1-\cos(\theta)}{3\theta}$
66. $\lim_{\theta \rightarrow 0} \frac{\sin^3(\theta)}{(2\theta)^3}$
67. $\lim_{\theta \rightarrow 0} \frac{4\theta^2+3\theta \sin(\theta)}{\theta^2}$
68. $\lim_{\theta \rightarrow 0} \frac{\sin[\cos(\theta)]}{\sec(\theta)}$
83. $\lim_{\theta \rightarrow 0} [\csc(\theta) - \cot(\theta)]$
84. $\lim_{\theta \rightarrow 0} \left[\frac{1}{\theta^2} - \frac{1}{\theta^2 \sec(\theta)} \right]$
85. $\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta+\theta^2}$
86. $\lim_{\theta \rightarrow 0} \frac{\sec(\theta)-1}{\theta^2}$
87. $\lim_{\theta \rightarrow 0} \frac{\cos(2\theta)-\cos(\theta)}{\sin^2(\theta)}$
88. $\lim_{\theta \rightarrow 0} \frac{\cos(\theta)}{\csc(\theta)}$
89. $\lim_{\theta \rightarrow 0} \frac{\theta^3}{\csc(\theta)+1}$
90. $\lim_{\theta \rightarrow 0} 2\theta^2 \sec^2(\theta) \cot^2(\theta)$
91. $\lim_{\theta \rightarrow 0} \frac{\cot^4(\theta) \tan(\theta) + \sin^2(\theta) - \csc(\theta) + \sec(\theta)}{\theta^{-3}}$
92. $\lim_{\theta \rightarrow 0} \left[3 \sec(\theta) - \frac{\theta^3 \csc^3(\theta)}{\cos^3(\theta)} + \theta^2 \csc(\theta) \right]$
93. $\lim_{\theta \rightarrow 0} [\cos(\theta) - \sin^3(\theta) \csc^2(\theta) - \tan(\theta)]$
94. $\lim_{\theta \rightarrow 0} [8\theta^2 \csc^2(\theta) + \tan(\theta) \cos(\theta)]$
95. $\lim_{\theta \rightarrow 0} 2\theta \cot(\theta) \sec(\theta)$
96. $\lim_{\theta \rightarrow 0} \left[\frac{\cot(\theta)}{\csc(\theta)} + \sec(\theta) \right]$
97. $\lim_{\theta \rightarrow 0} \frac{\sin(2\theta) \cos^3(\theta)}{\sin(\theta)}$
98. $\lim_{\theta \rightarrow 0} [\cos^2(\theta) - \sec(\theta) \sin(\theta)]$
99. $\lim_{\theta \rightarrow 0} \frac{\cot^2(\theta)+1}{\csc^2(\theta)}$
100. $\lim_{\theta \rightarrow 0} \frac{\csc^2(\theta)}{\cot^2(\theta)+1}$
101. $\lim_{\theta \rightarrow 0} \frac{\sec^2(\theta)}{\tan^2(\theta)+1}$

“Only he who never plays, never loses.”