The Weekly Rigor

No. 47 "A mathematician is a machine for turning coffee into theorems." May 16, 2015

101 Problems in Calculating Trigonometric Limits with Solutions

(Part 18)

98. $\lim_{\theta\to 0}\left[\cos^2(\theta)-\sec(\theta)\sin(\theta)\right]=\lim_{\theta\to 0}\left[\cos^2(\theta)-\frac{\sin(\theta)}{\cos(\theta)}\right]=\cos^2(0)-\frac{\sin(0)}{\cos(0)}=1-\frac{0}{1}=1.$

99. $\lim_{\theta\to 0}\frac{\cot^2(\theta)+1}{\csc^2(\theta)}=\lim_{\theta\to 0}\frac{\frac{\cos^2(\theta)}{\sin^2(\theta)}+1}{\frac{1}{\sin^2(\theta)}}=\lim_{\theta\to 0}\frac{\left(\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin^2(\theta)}\right)}{\left(\frac{1}{\sin^2(\theta)}\right)}=$

$=\lim_{\theta\to 0}\left[\cos^2(\theta)+\sin^2(\theta)\right]=\left[\cos^2(0)+\sin^2(0)\right]=1+0=1.$

Alternate solution:

$\lim_{\theta\to 0}\frac{\cot^2(\theta)+1}{\csc^2(\theta)}=\lim_{\theta\to 0}\frac{\csc^2(\theta)}{\csc^2(\theta)}=\lim_{\theta\to 0}1=1.$

100. $\lim_{\theta\to 0}\frac{\csc^2(\theta)}{\cot^2(\theta)+1}=\lim_{\theta\to 0}\frac{\frac{1}{\sin^2(\theta)}}{\frac{\cos^2(\theta)}{\sin^2(\theta)}+1}=\lim_{\theta\to 0}\frac{\frac{1}{\sin^2(\theta)}}{\frac{\cos^2(\theta)}{\sin^2(\theta)}+\frac{\sin^2(\theta)}{\sin^2(\theta)}}=$

$=\lim_{\theta\to 0}\frac{\left(\frac{1}{\sin^2(\theta)}\right)}{\left(\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin^2(\theta)}\right)}=\lim_{\theta\to 0}\frac{\left(\frac{1}{\sin^2(\theta)}\right)}{\left(\frac{1}{\sin^2(\theta)}\right)}=\lim_{\theta\to 0}1=1.$

Alternate solution:

$\lim_{\theta\to 0}\frac{\csc^2(\theta)}{\cot^2(\theta)+1}=\lim_{\theta\to 0}\frac{\csc^2(\theta)}{\csc^2(\theta)}=\lim_{\theta\to 0}1=1.$

101. $\lim_{\theta\to 0}\dfrac{\sec^{\,2}(\theta)}{\tan^{\,2}(\theta)+1}=\lim_{\theta\to 0}\dfrac{\dfrac{1}{\cos^{\,2}(\theta)}}{\dfrac{\sin^{\,2}(\theta)}{\cos^{\,2}(\theta)}+1}=\lim_{\theta\to 0}\dfrac{\dfrac{1}{\cos^{\,2}(\theta)}}{\dfrac{\sin^{\,2}(\theta)}{\cos^{\,2}(\theta)}+\dfrac{\cos^{\,2}(\theta)}{\cos^{\,2}(\theta)}}=$

$=\lim_{\theta\to 0}\dfrac{\left(\dfrac{1}{\cos^{\,2}(\theta)}\right)}{\left(\dfrac{\sin^{\,2}(\theta)+\cos^{\,2}(\theta)}{\cos^{\,2}(\theta)}\right)}=\lim_{\theta\to 0}\dfrac{\left(\dfrac{1}{\cos^{\,2}(\theta)}\right)}{\left(\dfrac{1}{\cos^{\,2}(\theta)}\right)}=\dfrac{\left(\dfrac{1}{\cos^{\,2}(0)}\right)}{\left(\dfrac{1}{\cos^{\,2}(0)}\right)}=\dfrac{\left(\frac{1}{1}\right)}{\left(\frac{1}{1}\right)}=1.$

Alternate solution:

$\lim_{\theta\to 0}\dfrac{\sec^{\,2}(\theta)}{\tan^{\,2}(\theta)+1}=\lim_{\theta\to 0}\dfrac{\sec^{\,2}(\theta)}{\sec^{\,2}(\theta)}=\lim_{\theta\to 0}1=1.$