The Weekly Rigor

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51 Problems in Calculating Integrals Using *U*-Substitution with Solutions

(Part 2)

In Type 4 problems, the procedure is

1. *u*-assignment
2. back substitution
3. *u*-substitution
4. integration in terms of *u*
5. *x*-substitution

Example: $\int (x+3)(x-1)^4\,dx$

1. *u*-assignment: $u = x - 1$
   $du = dx$

2. back substitution: $u = x - 1$
   $u + 4 = x + 3$

3. *u*-substitution: $\int (u+4)u^4\,du$

4. integration: $\int (u+4)u^4\,du = \int (u^5 + 4u^4)\,du = \int u^5\,du + 4\int u^4\,du = \frac{1}{6}u^6 + \frac{4}{5}u^5 + C$

5. *x*-substitution: $\frac{1}{6}(x-1)^6 + \frac{4}{5}(x-1)^5 + C$

Every solution in this articles has a "check step." This optional step verifies that the answer to the integral problem is correct by finding that the derivative of the "answer" equals the original integrand. For example, if we find that

$$\int (x+1)^4\,dx = \frac{1}{5}(x+1)^5 + C,$$

we can check this by:

$$\left[\frac{1}{5}(x+1)^5 + C\right]' = \frac{5}{5}(x+1)^4 = (x+1)^4\,.\ \checkmark$$

After the solutions, there is an appendix. This appendix includes two sets of randomly-ordered problems with answer keys following them. These problems are designed to help the student test his skill and detect which types of problems need special attention. The answers immediately following the problems in random order include the number of the original problem. The first set contains problems only from Types 1-3, whereas the second set contains problems from all four types. I broke up the sets this way since many calculus classes do not include Type 4 *u*-substitution problems.