

The Weekly Rigor

No. 53

“A mathematician is a machine for turning coffee into theorems.”

June 27, 2015

51 Problems in Calculating Integrals Using U -Substitution with Solutions (Part 4)

ANSWERS

Type 1

1. $\frac{1}{5}(x+1)^5 + C$

2. $\frac{1}{7}(x-50)^7 + C$

3. $\frac{2}{3}(x+1)^{\frac{3}{2}} + C$

4. $\frac{6}{7}(x-50)^{\frac{7}{6}} + C$

5. $\frac{-1}{2(x+2)^2} + C$

6. $\frac{-1}{4(x-21)^4} + C$

7. $2\sqrt{x+2} + C$

8. $\frac{5}{4}(x-21)^{\frac{4}{5}} + C$

9. $\ln|x+3| + C$

10. $\ln|x-3| + C$

11. $\sin(x+\pi) + C$

12. $-\cos(x-5) + C$

13. $e^{x+3} + C$

14. $e^{31+x} + C$

Type 2

15. $\frac{1}{15}(3x+1)^5 + C$

16. $\frac{2}{7}\left(\frac{1}{2}x-50\right)^7 + C$

17. $\frac{2}{9}\sqrt{(3x+1)^3} + C$

18. $\frac{12}{7}\sqrt[6]{\left(\frac{1}{2}x-50\right)^7} + C$

19. $\frac{-1}{6(3x+2)^2} + C$

20. $\frac{-1}{3}\left(\frac{3}{4}x-21\right)^{-4} + C$

21. $\frac{2}{3}\sqrt{3x+2} + C$

22. $\frac{35}{12}\sqrt[5]{\left(\frac{3}{7}x-21\right)^4} + C$

23. $\ln\sqrt{2x+3} + C$

24. $\ln\sqrt{\left(\frac{2}{5}x-3\right)^5} + C$

25. $\frac{1}{4}\sin(4x) + C$

26. $3\tan\left(\frac{1}{3}x\right) + C$

27. $\frac{1}{2}e^{2x+3} + C$

Type 3

$$28. \frac{1}{30}(3x^2 + 1)^5 + C$$

$$29. \frac{2}{21}\left(\frac{1}{2}x^3 - 50\right)^7 + C$$

$$30. \frac{4}{7}\left(\frac{1}{2}x^3 - 50\right)^{\frac{7}{6}} + C$$

$$31. \frac{-1}{12}(3x^2 + 2)^{-2} + C$$

$$32. \frac{1}{3}\sqrt{3x^2 + 2} + C$$

$$33. \frac{35}{36}\left(\frac{3}{7}x^3 - 21\right)^{\frac{4}{5}} + C$$

$$34. \frac{1}{4}\ln(2x^2 + 3) + C$$

$$35. \frac{5}{6}\ln\left|\frac{2}{5}x^3 - 3\right| + C$$

$$36. \frac{1}{6}\sin(3x^2) + C$$

$$37. -\frac{1}{2}\cos\left(\frac{2}{3}x^3 - 5\right) + C$$

$$38. \frac{-1}{4}(x^2 - 4x + 3)^{-2} + C$$

$$39. \frac{1}{2}e^{x^2} + C$$

$$40. \frac{1}{9}(x^3 + 3x)^3 + C$$

$$41. \frac{1}{2}\sin^2(x) + C$$

$$42. \ln|\sin(x)| + C$$

$$\text{or: } -\frac{1}{2}\cos^2(x) + C$$

Type 4

$$43. \frac{1}{6}(x - 1)^6 + \frac{4}{5}(x - 1)^5 + C$$

$$44. \frac{5}{32}(1 + x^2)^{\frac{16}{5}} - \frac{5}{11}(1 + x^2)^{\frac{11}{5}} + \frac{5}{12}(1 + x^2)^{\frac{6}{5}} + C$$

$$45. \frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + C$$

$$46. \frac{1}{6}(1 + 2x)^{\frac{3}{2}} - \frac{1}{2}(1 + 2x)^{\frac{1}{2}} + C$$

$$47. \frac{4}{7}(x + 2)^{\frac{7}{4}} - \frac{8}{3}(x + 2)^{\frac{3}{4}} + C$$

$$48. \frac{1}{4}(2x + 5) + \frac{3}{4}\ln|2x + 5| + C$$

$$49. \frac{1}{2}(x + 2)^2 - 4(x + 2) + 8\ln|x + 2| + C$$

$$50. \frac{1}{18}(x^3 + 1)^6 - \frac{1}{15}(x^3 + 1)^5 + C$$

$$51. \frac{5}{3}(3 + \ln(x))^3 - \frac{1}{4}(3 + \ln(x))^4 + C$$

$$\text{or: } \frac{-25}{2}(2 - \ln(x))^2 + \frac{10}{3}(2 - \ln(x))^3 - \frac{1}{4}(2 - \ln(x))^4 + C$$