The Weekly Rigor

No. 54 "A mathematician is a machine for turning coffee into theorems." July 4, 2015

51 Problems in Calculating Integrals Using *U*-Substitution with Solutions
(Part 5)

SOLUTIONS

Type 1

1. $\int (x+1)^4\,dx = \int u^4\,du = \frac{1}{5}u^5 + C = \frac{1}{5}(x+1)^5 + C.$ $\begin{cases} \text{Let } u = x+1 \\ du = dx \end{cases}$

Check: $\left[\frac{1}{5}(x+1)^5 + C\right]' = \frac{5}{5}(x+1)^4 = (x+1)^4.$ ✓

2. $\int (x-50)^6\,dx = \int u^6\,du = \frac{1}{7}u^7 + C = \frac{1}{7}(x-50)^7 + C.$ $\begin{cases} \text{Let } u = x-50 \\ du = dx \end{cases}$

Check: $\left[\frac{1}{7}(x-50)^7 + C\right]' = \frac{7}{7}(x-50)^6 = (x-50)^6.$ ✓

3. $\int \sqrt{x+1}\,dx = \int (x+1)^{\frac{1}{2}}\,dx = \int u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(x+1)^{\frac{3}{2}} + C.$ $\begin{cases} \text{Let } u = x+1 \\ du = dx \end{cases}$

Check: $\left[\frac{2}{3}(x+1)^{\frac{3}{2}} + C\right]' = \frac{2}{3}\cdot\frac{3}{2}(x+1)^{\frac{1}{2}} = (x+1)^{\frac{1}{2}} = \sqrt{x+1}.$ ✓

4. $\int \sqrt[6]{x-50}\,dx = \int (x-50)^{\frac{1}{6}}\,dx = \int u^{\frac{1}{6}}\,du = \frac{6}{7}u^{\frac{7}{6}} + C =$ $\begin{cases} \text{Let } u = x-50 \\ du = dx \end{cases}$

$= \frac{6}{7}(x-50)^{\frac{7}{6}} + C.$

Check: $\left[\frac{6}{7}(x-50)^{\frac{7}{6}} + C\right]' = \frac{6}{7}\cdot\frac{7}{6}(x-50)^{\frac{1}{6}} = (x-50)^{\frac{1}{6}} = \sqrt[6]{x-50}.$ ✓

5. $\int \frac{1}{(x+2)^3}\,dx = \int (x+2)^{-3}\,dx = \int u^{-3}\,du = -\frac{1}{2}u^{-2} + C =$ $\begin{cases} \text{Let } u = x+2 \\ du = dx \end{cases}$

$= -\frac{1}{2}(x+2)^{-2} + C = \frac{-1}{2(x+2)^2} + C.$

Check: $\left[\frac{-1}{2(x+2)^2} + C\right]' = \left[-\frac{1}{2}(x+2)^{-2} + C\right]' = -\frac{-2}{2}(x+2)^3 =$

$= (x+2)^{-3} = \frac{1}{(x+2)^3}.$ ✓

6. $\int \frac{1}{(x-21)^5}\,dx = \int (x-21)^{-5}\,dx = \int u^{-5}\,du = -\frac{1}{4}u^{-4} + C =$ 　 Let $u = x - 21$, $du = dx$

$-\frac{1}{4}(x-21)^{-4} + C = \frac{-1}{4(x-21)^4} + C.$

Check: $\left[\frac{-1}{4(x-21)^4} + C\right]' = \left[-\frac{1}{4}(x-21)^{-4} + C\right]' = -\frac{-4}{4}(x-21)^{-5} =$

$= (x-21)^{-5} = \frac{1}{(x-21)^5}$. ✓

7. $\int \frac{1}{\sqrt{x+2}}\,dx = \int \frac{1}{(x+2)^{\frac{1}{2}}}\,dx = \int (x+2)^{-\frac{1}{2}}\,dx = \int u^{-\frac{1}{2}}\,du =$ 　 Let $u = x + 2$, $du = dx$

$= 2u^{\frac{1}{2}} + C = 2(x+2)^{\frac{1}{2}} + C = 2\sqrt{x+2} + C.$

Check: $\left[2\sqrt{x+2} + C\right]' = \left[2(x+2)^{\frac{1}{2}} + C\right]' = \frac{2}{2}(x+2)^{-\frac{1}{2}} = (x+2)^{-\frac{1}{2}} =$

$= \frac{1}{(x+2)^{\frac{1}{2}}} = \frac{1}{\sqrt{x+2}}$. ✓

8. $\int \frac{1}{\sqrt[5]{x-21}}\,dx = \int \frac{1}{(x-21)^{\frac{1}{5}}}\,dx = \int (x-21)^{-\frac{1}{5}}\,dx = \int u^{-\frac{1}{5}}\,du =$ 　 Let $u = x - 21$, $du = dx$

$= \frac{5}{4}u^{\frac{4}{5}} + C = \frac{5}{4}(x-21)^{\frac{4}{5}} + C.$

Check: $\left[\frac{5}{4}(x-21)^{\frac{4}{5}} + C\right]' = \frac{5}{4}\cdot\frac{4}{5}(x-21)^{-\frac{1}{5}} = (x-21)^{-\frac{1}{5}} = \frac{1}{\sqrt[5]{x-21}}$. ✓

9. $\int \frac{1}{x+3}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x+3| + C.$ 　 Let $u = x + 3$, $du = dx$

Check: $[\ln|x+3| + C]' = \frac{1}{x+3}$. ✓

10. $\int \frac{1}{x-3}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x-3| + C.$ 　 Let $u = x - 3$, $du = dx$

Check: $[\ln|x-3| + C]' = \frac{1}{x-3}$. ✓

11. $\int \cos(x+\pi)\,dx = \int \cos(u)\,du = \sin(u) + C = \sin(x+\pi) + C.$ 　 Let $u = x + \pi$, $du = dx$

Check: $[\sin(x+\pi) + C]' = \cos(x+\pi)$. ✓

12. $\int \sin(x-5)\,dx = \int \sin(u)\,du = -\cos(u) + C = -\cos(x-5) + C.$ 　 Let $u = x - 5$, $du = dx$

Check: $[-\cos(x-5) + C]' = -[-\sin(x-5)] = \sin(x-5)$. ✓