The Weekly Rigor

No. 55 "A mathematician is a machine for turning coffee into theorems." July 11, 2015

51 Problems in Calculating Integrals Using *U*-Substitution with Solutions
(Part 6)

13. $\int e^{x+3}\,dx = \int e^u\,du = e^u + C = e^{x+3} + C.$ $\left\{\begin{aligned}\text{Let } u &= x+3\\ du &= dx\end{aligned}\right.$

Check: $[e^{x+3} + C]' = e^{x+3}$. ✓

Type 2

14. $\int e^{31+x}\,dx = \int e^u\,du = e^u + C = e^{31+x} + C.$ $\left\{\begin{aligned}\text{Let } u &= 31+x\\ du &= dx\end{aligned}\right.$

Check: $[e^{31+x} + C]' = e^{31+x}$. ✓

15. $\int (3x+1)^4\,dx = \int u^4 \frac{du}{3} = \frac{1}{3}\int u^4 du = \frac{1}{3}\cdot\frac{1}{5}u^5 + C = \frac{1}{15}u^5 + C =$ $\left\{\begin{aligned}\text{Let } u &= 3x+1\\ du &= 3dx\\ \tfrac{du}{3} &= dx\end{aligned}\right.$

$= \frac{1}{15}(3x+1)^5 + C.$

Check: $\left[\frac{1}{15}(3x+1)^5 + C\right]' = \frac{5}{15}(3x+1)^4(3) = (3x+1)^4$. ✓

Remark: Some books and teachers would do the *u*-assignment step differently as follows:

Let $u = 3x + 1$

$du = 3dx$

and then modify the original integral:

$$\int (3x+1)^4\,dx = \frac{1}{3}\int (3x+1)^4 3\,dx,$$

which will result in the following *u*-substitution:

$$\frac{1}{3}\int u^4\,du.$$

In over twenty years of tutoring calculus, every student with whom I discussed this alternative way of doing the problem *hates* it. I agree. I think the *u*-assignment technique employed in this book makes the *u*-substitution step as "obvious" and "mechanical" as can be.

16. $\int \left(\frac{1}{2}x - 50\right)^6 dx = \int u^6\, 2du = 2\int u^6 du = 2 \cdot \frac{1}{7}u^7 + C = \frac{2}{7}u^7 + C =$ $\left\{\begin{array}{l}\text{Let } u = \frac{1}{2}x - 50 \\ du = \frac{1}{2}dx \\ 2du = dx\end{array}\right.$

$= \frac{2}{7}\left(\frac{1}{2}x - 50\right)^7 + C.$

Check: $\left[\frac{2}{7}\left(\frac{1}{2}x - 50\right)^7 + C\right]' = \frac{14}{7}\left(\frac{1}{2}x - 50\right)^6\left(\frac{1}{2}\right) = \left(\frac{1}{2}x - 50\right)^6.$ ✓

17. $\int \sqrt{3x+1}\, dx = \int (3x+1)^{\frac{1}{2}} dx = \int u^{\frac{1}{2}} \frac{du}{3} = \frac{1}{3}\int u^{\frac{1}{2}} du = \frac{1}{3}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C =$ $\left\{\begin{array}{l}\text{Let } u = 3x + 1 \\ du = 3dx \\ \frac{du}{3} = dx\end{array}\right.$

$= \frac{1}{3} \cdot \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{9}u^{\frac{3}{2}} + C = \frac{2}{9}(3x+1)^{\frac{3}{2}} + C = \frac{2}{9}\sqrt{(3x+1)^3} + C.$

Check: $\left[\frac{2}{9}\sqrt{(3x+1)^3} + C\right]' = \left[\frac{2}{9}(3x+1)^{\frac{3}{2}} + C\right]' = \frac{2}{9} \cdot \frac{3}{2}(3x+1)^{\frac{1}{2}}(3) =$

$= (3x+1)^{\frac{1}{2}} = \sqrt{3x+1}.$ ✓

18. $\int \sqrt[6]{\frac{1}{2}x - 50}\, dx = \int \left(\frac{1}{2}x - 50\right)^{\frac{1}{6}} dx = \int u^{\frac{1}{6}} 2du = 2\int u^{\frac{1}{6}} du = 2\frac{u^{\frac{7}{6}}}{\left(\frac{7}{6}\right)} + C =$ $\left\{\begin{array}{l}\text{Let } u = \frac{1}{2}x - 50 \\ du = \frac{1}{2}dx \\ 2du = dx\end{array}\right.$

$= 2 \cdot \frac{6}{7}u^{\frac{7}{6}} + C = \frac{12}{7}u^{\frac{7}{6}} + C = \frac{12}{7}\left(\frac{1}{2}x - 50\right)^{\frac{7}{6}} + C = \frac{12}{7}\sqrt[6]{\left(\frac{1}{2}x - 50\right)^7} + C.$

Check: $\left[\frac{12}{7}\sqrt[6]{\left(\frac{1}{2}x - 50\right)^7} + C\right]' = \left[\frac{12}{7}\left(\frac{1}{2}x - 50\right)^{\frac{7}{6}} + C\right]' =$

$= \frac{12}{7} \cdot \frac{7}{6}\left(\frac{1}{2}x - 50\right)^{\frac{1}{6}} = 2\left(\frac{1}{2}x - 50\right)^{\frac{1}{6}}\left(\frac{1}{2}\right) = \left(\frac{1}{2}x - 50\right)^{\frac{1}{6}} = \sqrt[6]{\frac{1}{2}x - 50}.$ ✓

19. $\int \frac{1}{(3x+2)^3} dx = \int (3x+2)^{-3} dx = \int u^{-3} \frac{du}{3} = \frac{1}{3}\int u^{-3} du =$ $\left\{\begin{array}{l}\text{Let } u = 3x + 2 \\ du = 3dx \\ \frac{du}{3} = dx\end{array}\right.$

$= \frac{1}{3} \cdot \frac{-1}{2}u^{-2} + C = \frac{-1}{6}u^{-2} + C = \frac{-1}{6}(3x+2)^{-2} + C = \frac{-1}{6(3x+2)^2} + C.$

Check: $\left[\frac{-1}{6(3x+2)^2} + C\right]' = \left[\frac{-1}{6}(3x+2)^{-2} + C\right]' = \frac{2}{6}(3x+2)^3(3) =$

$= \frac{1}{3}(3x+2)^{-3}(3) = (3x+2)^{-3} = \frac{1}{(3x+2)^3}.$ ✓