The Weekly Rigor

No. 56 "A mathematician is a machine for turning coffee into theorems." July 18, 2015

51 Problems in Calculating Integrals Using *U*-Substitution with Solutions

(Part 7)

20. $\int \frac{1}{\left(\frac{3}{4}x-21\right)^5}dx = \int \left(\frac{3}{4}x - 21\right)^{-5} dx = \int u^{-5}\frac{4}{3}du = \frac{4}{3}\int u^{-5}du =$

$= \frac{4}{3}\cdot\frac{-1}{4}u^{-4} + C = \frac{-1}{3}u^{-4} + C = \frac{-1}{3}\left(\frac{3}{4}x - 21\right)^{-4} + C.$

$\left\{ \begin{array}{l} \text{Let } u = \frac{3}{4}x - 21 \\ du = \frac{3}{4}dx \\ \frac{4}{3}du = dx \end{array} \right.$

Check: : $\left[\frac{-1}{3}\left(\frac{3}{4}x - 21\right)^{-4} + C\right]' = \frac{4}{3}\left(\frac{3}{4}x - 21\right)^{-5}\frac{3}{4} = \left(\frac{3}{4}x - 21\right)^{-5} =$

$= \frac{1}{\left(\frac{3}{4}x-21\right)^5}$. ✓

21. $\int \frac{1}{\sqrt{3x+2}}dx = \int \frac{1}{(3x+2)^{\frac{1}{2}}}dx = \int (3x + 2)^{-\frac{1}{2}}dx = \int u^{-\frac{1}{2}}\frac{du}{3} =$

$= \frac{1}{3}\int u^{-\frac{1}{2}}du = \frac{1}{3}\cdot\frac{2}{1}u^{\frac{1}{2}} + C = \frac{2}{3}u^{\frac{1}{2}} + C = \frac{2}{3}(3x + 2)^{\frac{1}{2}} + C =$

$= \frac{2}{3}\sqrt{3x + 2} + C.$

$\left\{ \begin{array}{l} \text{Let } u = 3x + 2 \\ du = 3dx \\ \frac{du}{3} = dx \end{array} \right.$

Check: $\left[\frac{2}{3}\sqrt{3x + 2} + C\right]' = \left[\frac{2}{3}(3x + 2)^{\frac{1}{2}} + C\right]' =$

$= \frac{2}{3}\cdot\frac{1}{2}(3x + 2)^{-\frac{1}{2}}(3) = \frac{1}{3}(3x + 2)^{-\frac{1}{2}}(3) = (3x + 2)^{-\frac{1}{2}} = \frac{1}{(3x+2)^{\frac{1}{2}}} =$

$= \frac{1}{\sqrt{3x+2}}$. ✓

22. $\int \frac{1}{\sqrt[5]{\frac{3}{7}x-21}}dx = \int \frac{1}{(\frac{3}{7}x-21)^{\frac{1}{5}}}dx = \int \left(\frac{3}{7}x - 21\right)^{-\frac{1}{5}} dx = \int u^{-\frac{1}{5}}\frac{7}{3}du =$

$= \frac{7}{3}\int u^{-\frac{1}{5}}du = \frac{7}{3}\cdot\frac{5}{4}u^{\frac{4}{5}} + C = \frac{35}{12}u^{\frac{4}{5}} + C = \frac{35}{12}\left(\frac{3}{7}x - 21\right)^{\frac{4}{5}} + C =$

$= \frac{35}{12}\sqrt[5]{\left(\frac{3}{7}x - 21\right)^4} + C.$

Let $u = \frac{3}{7}x - 21$
$du = \frac{3}{7}dx$
$\frac{7}{3}du = dx$

Check: $\left[\frac{35}{12}\sqrt[5]{\left(\frac{3}{7}x - 21\right)^4} + C\right]' = \left[\frac{35}{12}\left(\frac{3}{7}x - 21\right)^{\frac{4}{5}} + C\right]' =$

$= \frac{35}{12}\cdot\frac{4}{5}\left(\frac{3}{7}x - 21\right)^{-\frac{1}{5}}\frac{3}{7} = \left(\frac{3}{7}x - 21\right)^{-\frac{1}{5}} = \frac{1}{(\frac{3}{7}x-21)^{\frac{1}{5}}} = \frac{1}{\sqrt[5]{\frac{3}{7}x-21}}$. ✓

23. $\int \frac{1}{2x+3}dx = \int \frac{1}{u}\frac{du}{2} = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x + 3| + C =$

$= \ln(2x + 3)^{\frac{1}{2}} + C = \ln\sqrt{2x + 3} + C.$

Let $u = 2x + 3$
$du = 2dx$
$\frac{du}{2} = dx$

Check: $\left[\ln\sqrt{2x + 3} + C\right]' = \left[\ln(2x + 3)^{\frac{1}{2}} + C\right]' =$

$= \left[\frac{1}{2}\ln|2x + 3| + C\right]' = \frac{1}{2}\cdot\frac{2}{2x+3} = \frac{1}{2x+3}$. ✓

24. $\int \frac{1}{\frac{2}{5}x-3}dx = \int \frac{1}{u}\cdot\frac{5}{2}du = \frac{5}{2}\int \frac{1}{u}du = \frac{5}{2}\ln|u| + C = \frac{5}{2}\ln\left|\frac{2}{5}x - 3\right| + C =$

$= \ln\left(\frac{2}{5}x - 3\right)^{\frac{5}{2}} + C = \ln\sqrt{\left(\frac{2}{5}x - 3\right)^5} + C.$

Let $u = \frac{2}{5}x - 3$
$du = \frac{2}{5}dx$
$\frac{5}{2}du = dx$

Check: $\left[\ln\sqrt{\left(\frac{2}{5}x - 3\right)^5} + C\right]' = \left[\ln\left(\frac{2}{5}x - 3\right)^{\frac{5}{2}} + C\right]' =$

$= \left[\frac{5}{2}\ln\left|\frac{2}{5}x - 3\right| + C\right]' = \frac{5}{2}\cdot\frac{\frac{2}{5}}{\frac{2}{5}x-3} = \frac{5}{2}\cdot\frac{2}{5}\cdot\frac{1}{\frac{2}{5}x-3} = \frac{1}{\frac{2}{5}x-3}$. ✓

25. $\int \cos(4x)\,dx = \int \cos(u)\frac{du}{4} = \frac{1}{4}\int \cos(u)\,du = \frac{1}{4}\sin(u) + C =$

$= \frac{1}{4}\sin(4x) + C.$

Let $u = 4x$
$du = 4dx$
$\frac{du}{4} = dx$

Check: $\left[\frac{1}{4}\sin(4x) + C\right]' = \frac{1}{4}\cos(4x)\cdot 4 = \cos(4x)$. ✓