The Weekly Rigor

No. 58 "A mathematician is a machine for turning coffee into theorems." August 1, 2015

51 Problems in Calculating Integrals Using *U*-Substitution with Solutions

(Part 9)

34. $\int \frac{x}{2x^2+3}dx = \int \frac{1}{2x^2+3}x\,dx = \int \frac{1}{u}\frac{du}{4} = \frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln|u| + C =$
$= \frac{1}{4}\ln(2x^2 + 3) + C.$

$\begin{cases} \text{Let } u = 2x^2 + 3 \\ du = 4xdx \\ \frac{du}{4} = xdx \end{cases}$

Remark: The absolute value bars can be replaced by parentheses since $2x^2 + 3$ is greater than zero for all x.

Check: $\left[\frac{1}{4}\ln(2x^2 + 3) + C\right]' = \frac{1}{4}\cdot\frac{4x}{2x^2+3} = \frac{x}{2x^2+3}$. ✓

35. $\int \frac{x^2}{\frac{2}{5}x^3-3}dx = \int \frac{1}{\frac{2}{5}x^3-3}x^2dx = \int \frac{1}{u}\frac{5}{6}du = \frac{5}{6}\int \frac{1}{u}du = \frac{5}{6}\ln|u| + C =$
$= \frac{5}{6}\ln\left|\frac{2}{5}x^3 - 3\right| + C.$

$\begin{cases} \text{Let } u = \frac{2}{5}x^3 - 3 \\ du = \frac{6}{5}x^2dx \\ \frac{5}{6}du = x^2dx \end{cases}$

Check: $\left[\frac{5}{6}\ln\left|\frac{2}{5}x^3 - 3\right| + C\right]' = \frac{5}{6}\cdot\frac{\frac{6}{5}x^2}{\frac{2}{5}x^3-3} = \frac{5}{6}\cdot\frac{6}{5}\cdot\frac{x^2}{\frac{2}{5}x^3-3} = \frac{x^2}{\frac{2}{5}x^3-3}$. ✓

36. $\int x\cos(3x^2)\,dx = \int \cos(3x^2)x\,dx = \int \cos(u)\frac{du}{6} = \frac{1}{6}\int \cos(u)\,du =$
$= \frac{1}{6}\sin(u) + C = \frac{1}{6}\sin(3x^2) + C.$

$\begin{cases} \text{Let } u = 3x^2 \\ du = 6xdx \\ \frac{du}{6} = xdx \end{cases}$

Check: $\left[\frac{1}{6}\sin(3x^2) + C\right]' = \frac{1}{6}\cos(3x^2)\cdot 6x = x\cos(3x^2)$. ✓

37. $\int x^2 \sin\left(\frac{2}{3}x^3 - 5\right) dx = \int \sin\left(\frac{2}{3}x^3 - 5\right) x^2\, dx = \int \sin(u) \frac{du}{2} =$

$= \frac{1}{2}\int \sin(u)\, du = \frac{1}{2} \cdot [-\cos(u)] + C = -\frac{1}{2}\cos(u) + C =$

$= -\frac{1}{2}\cos\left(\frac{2}{3}x^3 - 5\right) + C.$

Check: $\left[-\frac{1}{2}\cos\left(\frac{2}{3}x^3 - 5\right) + C\right]' = \frac{1}{2}\sin\left(\frac{2}{3}x^3 - 5\right) \cdot 2x^2 =$

$= x^2 \sin\left(\frac{2}{3}x^3 - 5\right)$. ✓

Let $u = \frac{2}{3}x^3 - 5$
$du = 2x^2 dx$
$\frac{du}{2} = x^2 dx$

38. $\int \frac{x-2}{(x^2-4x+3)^3} dx = \int \frac{1}{(x^2-4x+3)^3}(x-2)\, dx =$

$= \int \frac{1}{u^3}\frac{du}{2} = \frac{1}{2}\int \frac{1}{u^3} du = \frac{1}{2}\int u^{-3}\, du = \frac{1}{2} \cdot \frac{-1}{2} u^{-2} + C =$

$= \frac{-1}{4} u^{-2} + C = \frac{-1}{4}(x^2 - 4x + 3)^{-2} + C.$

Check: $\left[\frac{-1}{4}(x^2 - 4x + 3)^{-2} + C\right]' = \frac{1}{2}(x^2 - 4x + 3)^{-3}(2x - 4) =$

$= \frac{1}{2}(x^2 - 4x + 3)^{-3} 2(x-2) = (x^2 - 4x + 3)^{-3}(x - 2) =$

$= \frac{x-2}{(x^2-4x+3)^3}$. ✓

Let $u = x^2 - 4x + 3$
$du = (2x - 4)dx$
$du = 2(x - 2)dx$
$\frac{du}{2} = (x - 2)dx$

39. $\int e^{x^2} x\, dx = \int e^u \frac{du}{2} = \frac{1}{2}\int e^u\, du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C.$

Check: $\left[\frac{1}{2}e^{x^2} + C\right]' = \frac{1}{2}e^{x^2} 2x = e^{x^2} x$. ✓

Let $u = x^2$
$du = 2xdx$
$\frac{du}{2} = xdx$

40. $\int (x^3 + 3x)^2 (x^2 + 1)\, dx = \int u^2 \frac{du}{3} = \frac{1}{3}\int u^2\, du = \frac{1}{3} \cdot \frac{1}{3}u^3 + C =$

$= \frac{1}{9}u^3 + C = \frac{1}{9}(x^3 + 3x)^3 + C.$

Check: $\left[\frac{1}{9}(x^3 + 3x)^3 + C\right]' = \frac{3}{9}(x^3 + 3x)^2(3x^2 + 3) =$

$= \frac{1}{3}(x^3 + 3x)^2(3x^2 + 3) = \frac{1}{3}(x^3 + 3x)^2 3(x^2 + 1)$

$= (x^3 + 3x)^2(x^2 + 1)$. ✓

Let $u = x^3 + 3x$
$du = (3x^2 + 3)dx$
$du = 3(x^2 + 1)dx$
$\frac{du}{3} = (x^2 + 1)dx$