The Weekly Rigor

No. 63 "A mathematician is a machine for turning coffee into theorems." September 5, 2015

51 Problems in Calculating Integrals Using *U*-Substitution with Solutions

(Part 14)

TYPE 1-4 PROBLEMS IN RANDOM ORDER

Calculate the following integrals.

1. $\int (3x+1)^4\,dx$
2. $\int \frac{1}{(x+2)^3}\,dx$
3. $\int \frac{1}{\left(\frac{3}{4}x-21\right)^5}\,dx$
4. $\int \sqrt{x+1}\,dx$
5. $\int \frac{1}{(3x+2)^3}\,dx$
6. $\int (x+1)^4\,dx$
7. $\int \frac{1}{2x+3}\,dx$
8. $\int \frac{x^2}{\sqrt[5]{\frac{3}{7}x^3-21}}\,dx$
9. $\int \cos(4x)\,dx$
10. $\int \frac{x}{\sqrt{3x^2+2}}\,dx$
11. $\int \cot(x)\,dx$
12. $\int \frac{(3+\ln(x))^2(2-\ln(x))}{x}\,dx$
13. $\int \frac{x^2}{\frac{2}{5}x^3-3}\,dx$
14. $\int (x+3)(x-1)^4\,dx$
15. $\int \left(\frac{1}{2}x-50\right)^6\,dx$
16. $\int \frac{1}{\sqrt[5]{x-21}}\,dx$
17. $\int e^{31+x}\,dx$
18. $\int \frac{1}{(x-21)^5}\,dx$
19. $\int \frac{1}{x+3}\,dx$
20. $\int \frac{1}{\sqrt{3x+2}}\,dx$
21. $\int x^2\sqrt[6]{\frac{1}{2}x^3-50}\,dx$
22. $\int (3x^2+1)^4 x\,dx$
23. $\int \sin(x)\cos(x)\,dx$
24. $\int \frac{x+4}{2x+5}\,dx$
25. $\int (x^3+1)^4 x^5\,dx$
26. $\int x\cos(3x^2)\,dx$
27. $\int \frac{x}{\sqrt{1+2x}}\,dx$
28. $\int \cos(x+\pi)\,dx$
29. $\int \sin(x-5)\,dx$
30. $\int \frac{1}{x-3}\,dx$
31. $\int e^{x+3}\,dx$
32. $\int \frac{1}{\sqrt{x+2}}\,dx$
33. $\int \frac{1}{\sqrt[5]{\frac{3}{7}x-21}}\,dx$
34. $\int \left(\frac{1}{2}x^3-50\right)^6 x^2\,dx$
35. $\int \frac{1}{\frac{2}{5}x-3}\,dx$
36. $\int \frac{x}{2x^2+3}\,dx$
37. $\int \sec^2\left(\frac{1}{3}x\right)dx$
38. $\int \frac{x^2+4}{x+2}\,dx$
39. $\int \frac{(3+\ln(x))^2(2-\ln(x))}{x}\,dx$
40. $\int \frac{x-2}{(x^2-4x+3)^3}\,dx$
41. $\int \frac{x}{\sqrt[4]{x+2}}\,dx$
42. $\int e^{x^2}x\,dx$
43. $\int \sqrt[6]{x-50}\,dx$
44. $\int \sqrt[6]{\frac{1}{2}x-50}\,dx$
45. $\int \sqrt{3x+1}\,dx$
46. 2. $\int (x-50)^6\,dx$
47. $\int \frac{x}{(3x^2+2)^3}\,dx$
48. $\int e^{2x+3}\,dx$
49. $\int x^5\sqrt[5]{1+x^2}\,dx$
50. $\int (x^3+3x)^2(x^2+1)\,dx$
51. $\int x\sqrt{x-1}\,dx$

ANSWERS

1. $\frac{1}{15}(3x+1)^5+C$ (#15)
2. $\frac{-1}{2(x+2)^2}++C$ (#5)
3. $\frac{-1}{3}\left(\frac{3}{4}x-21\right)^{-4}+C$ (#20)
4. $\frac{2}{3}(x+1)^{\frac{3}{2}}+C$ (#3)
5. $\frac{-1}{6(3x+2)^2}+C$ (#19)
6. $\frac{1}{5}(x+1)^5+C$ (#1)
7. $\ln\sqrt{2x+3}+C$ (#23)
8. $\frac{35}{36}\left(\frac{3}{7}x^3-21\right)^{\frac{4}{5}}+C$ (#33)
9. $\frac{1}{4}\sin(4x)+C$ (#25)
10. $\frac{1}{3}\sqrt{3x^2+2}+C$ (#32)
11. $\ln|\sin(x)|+C$ (#42)
12. $\frac{5}{3}(3+\ln(x))^3-\frac{1}{4}(3+\ln(x))^4+C$ (#51)
13. $\frac{5}{6}\ln\left|\frac{2}{5}x^3-3\right|+C$ (#35)
14. $\frac{1}{6}(x-1)^6+\frac{4}{5}(x-1)^5+C$ (#43)
15. $\frac{2}{7}\left(\frac{1}{2}x-50\right)^7+C$ (#16)
16. $\frac{5}{4}(x-21)^{\frac{4}{5}}+C$ (#8)
17. $e^{31+x}+C$ (#14)
18. $\frac{-1}{4(x-21)^4}+C$ (#6)
19. $\ln|x+3|+C$ (#9)
20. $\frac{2}{3}\sqrt{3x+2}+C$ (#21)
21. $\frac{4}{7}\left(\frac{1}{2}x^3-50\right)^{\frac{7}{6}}+C$ (#30)
22. $\frac{1}{30}(3x^2+1)^5+C$ (#28)
23. $\frac{1}{2}\sin^2(x)+C$ (#41)
24. $\frac{1}{4}(2x+5)+\frac{3}{4}\ln|2x+5|+C$ (#48)
25. $\frac{1}{18}(x^3+1)^6-\frac{1}{15}(x^3+1)^5+C$ (#50)
26. $\frac{1}{6}\sin(3x^2)+C$ (#36)
27. $\frac{1}{6}(1+2x)^{\frac{3}{2}}-\frac{1}{2}(1+2x)^{\frac{1}{2}}+C$ (#46)
28. $\sin(x+\pi)+C$ (#11)
29. $-\cos(x-5)+C$ (#12)
30. $\ln|x-3|+C$ (#10)
31. $e^{x+3}+C$ (#13)
32. $2\sqrt{x+2}+C$ (#7)
33. $\frac{35}{12}\sqrt[5]{\left(\frac{3}{7}x-21\right)^4}+C$ (#22)
34. $\frac{2}{21}\left(\frac{1}{2}x^3-50\right)^7+C$ (#29)
35. $\ln\sqrt{\left(\frac{2}{5}x-3\right)^5}+C$ (#24)
36. $\frac{1}{4}\ln(2x^2+3)+C$ (#34)
37. $3\tan\left(\frac{1}{3}x\right)+C$ (#26)
38. $\frac{1}{2}(x+2)^2-4(x+2)+8\ln|x+2|+C$ (#49)
39. $\frac{5}{3}(3+\ln(x))^3-\frac{1}{4}(3+\ln(x))^4+C$ (#51)
40. $\frac{-1}{4}(x^2-4x+3)^{-2}+C$ (#38)
41. $\frac{4}{7}(x+2)^{\frac{7}{4}}-\frac{8}{3}(x+2)^{\frac{3}{4}}+C$ (#47)
42. $\frac{1}{2}e^{x^2}+C$ (#39)
43. $\frac{6}{7}(x-50)^{\frac{7}{6}}+C$ (#4)
44. $\frac{12}{7}\sqrt[6]{\left(\frac{1}{2}x-50\right)^7}+C$ (#18)
45. $\frac{2}{9}\sqrt{(3x+1)^3}+C$ (#17)
46. $\frac{1}{7}(x-50)^7+C$ (#2)
47. $\frac{-1}{12}(3x^2+2)^{-2}+C$ (#31)
48. $\frac{1}{2}e^{2x+3}+C$ (#27)
49. $\frac{5}{32}(1+x^2)^{\frac{16}{5}}-\frac{5}{11}(1+x^2)^{\frac{11}{5}}+\frac{5}{12}(1+x^2)^{\frac{6}{5}}+C$ (#44)
50. $\frac{1}{9}(x^3+3x)^3+C$ (#40)
51. $\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+C$ (#45)