

The Weekly Rigor

No. 64

“A mathematician is a machine for turning coffee into theorems.”

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51 Problems in Calculating Derivatives Using the Chain Rule with Solutions (Part 1)

PROBLEMS

Find the derivative $f'(x)$ of each of the following functions.

- $f(x) = (x^2 + 3)^4$
- $f(x) = (x^3 + 5x^2 + 2)^5$
- $f(x) = (x^2 + 2)^{\frac{3}{2}}$
- $f(x) = (2x^3 + 3x^2 + 4)^{\frac{5}{3}}$
- $f(x) = \sqrt{x^2 + 3}$
- $f(x) = \sqrt[4]{x^3 + 3x^2 + 4}$
- $f(x) = \sqrt[3]{(x^2 + 4x)^2}$
- $f(x) = (\sqrt{x^3 + 4})^3$
- $f(x) = (x^2 + 2)^{-3}$
- $f(x) = (2x^3 + 3x^2 + 1)^{-\frac{1}{2}}$
- $f(x) = \frac{1}{(2x^5 + 6x^2)^3}$
- $f(x) = \frac{1}{(3x^4 + 2x)^{\frac{5}{6}}}$
- $f(x) = \frac{1}{\sqrt[3]{2x-1}}$
- $f(x) = \frac{1}{\sqrt{(2x^3 + 4x)^5}}$
- $f(x) = \left(\frac{x^2 + 3}{x + 1}\right)^3$
- $f(x) = \left(\frac{x-3}{x+8}\right)^4$
- $f(x) = \left(x^{\frac{2}{3}} + 3x^{\frac{1}{2}}\right)^3$
- $f(x) = (\sqrt{x} + 4\sqrt[3]{x^2})^{\frac{4}{3}}$
- $f(x) = \sqrt[5]{1 + x^{\frac{2}{3}}}$
- $f(x) = \sqrt[4]{3\sqrt{x} + 2x^{\frac{4}{3}}}$
- $f(x) = \left(2x^{\frac{3}{4}} + 5x^{\frac{1}{2}}\right)^{-4}$
- $f(x) = \frac{1}{(2\sqrt[3]{x} + 3x^2)^{-2}}$
- $f(x) = \frac{1}{\sqrt[4]{2\sqrt{x} + 5x}}$
- $f(x) = \frac{1}{\sqrt[5]{(3\sqrt{x} + 42x^{\frac{1}{3}})^2}}$
- $f(x) = (e^x + e^{-x})^3$
- $f(x) = \frac{1}{\sqrt{e^{2x} - 4e^{3x}}}$
- $f(x) = \ln^2(x)$
- $f(x) = (\ln(2x) + \ln(x))^{-3}$
- $f(x) = \sin^5(x)$
- $f(x) = \tan^3(x)$
- $f(x) = \arctan^2(x)$
- $f(x) = \arcsin^3(x)$
- $f(x) = e^{3x}$
- $f(x) = \ln(\sin(x))$
- $f(x) = \ln(1 + \sqrt{x})$
- $f(x) = \sin(3x)$
- $f(x) = \cos\left(\frac{x}{5}\right)$
- $f(x) = \tan(\sin(x))$
- $f(x) = \cos(x^3)$
- $f(x) = \arctan(3x)$
- $f(x) = \arccos(e^x)$
- $f(x) = (x^2 + 3)^4(x^2 + 2)^{\frac{3}{2}}$
- $f(x) = \sqrt{x^3 + 1}(x^2 + 1)^4$
- $f(x) = \sin(2x) \cos(3x)$
- $f(x) = e^{2x} \tan^3(x)$
- $f(x) = \frac{(x^2 + 3)^4}{(x^2 + 2)^{\frac{3}{2}}}$
- $f(x) = \frac{e^{2x}}{\sin(3x)}$
- $f(x) = \sin^2(3x)$
- $f(x) = e^{\cos(4x)}$
- $f(x) = \sin^3(\cos(2x))$
- $f(x) = \tan^4(\ln(e^{\sin(3x)}))$

ANSWERS

1. $f'(x) = 8x(x^2 + 3)^3$
2. $f'(x) = 5x(x^3 + 5x^2 + 2)^4(3x + 10)$
3. $f'(x) = 3x\sqrt{x^2 + 2}$
4. $f'(x) = 10x(x + 1)^3\sqrt{(2x^3 + 3x^2 + 4)^2}$
5. $f'(x) = \frac{x}{\sqrt{x^2+3}}$
6. $f'(x) = \frac{3x(x+2)}{4\sqrt[4]{(x^3+3x^2+4)^3}}$
7. $f'(x) = \frac{4(x+2)}{3\sqrt[3]{x^2+4x}}$
8. $f'(x) = \frac{9}{2}x^2\sqrt{x^3 + 4}$
9. $f'(x) = \frac{-6x}{(x^2+2)^4}$
10. $f'(x) = \frac{-3x(x+1)}{\sqrt{(2x^3+3x^2+1)^3}}$
11. $f'(x) = \frac{-6x(5x^3+6)}{(2x^5+6x^2)^4}$
12. $f'(x) = \frac{-5(6x^3+1)}{3\sqrt[6]{(3x^4+2x)^{11}}}$
13. $f'(x) = \frac{-2}{3\sqrt[3]{(2x-1)^4}}$
14. $f'(x) = \frac{-5(3x^2+2)}{\sqrt{(2x^3+4x)^7}}$
15. $f'(x) = \frac{3(x^2+3)^2(x+3)(x-1)}{(x+1)^4}$
16. $f'(x) = \frac{44(x-3)^3}{(x+8)^5}$
17. $f'(x) = 3\left(x^{\frac{2}{3}} + 3x^{\frac{1}{2}}\right)^2\left(\frac{2}{3}x^{-\frac{1}{3}} + \frac{3}{2}x^{-\frac{1}{2}}\right)$
18. $f'(x) = \frac{4}{3}\left(x^{\frac{1}{2}} + 4x^{\frac{2}{3}}\right)^{\frac{1}{3}}\left(\frac{1}{2}x^{-\frac{1}{2}} + \frac{8}{3}x^{-\frac{1}{3}}\right)$
19. $f'(x) = \frac{2}{15}x^{-\frac{1}{3}}\left(1 + x^{\frac{2}{3}}\right)^{\frac{4}{5}}$
20. $f'(x) = \frac{1}{4}\left(3x^{\frac{1}{2}} + 2x^{\frac{4}{3}}\right)^{-\frac{3}{4}}\left(\frac{3}{2}x^{-\frac{1}{2}} + \frac{8}{3}x^{\frac{1}{3}}\right)$
21. $f'(x) = -4\left(2x^{\frac{3}{4}} + 5x^{\frac{1}{2}}\right)^{-5}\left(\frac{3}{2}x^{\frac{1}{4}} + \frac{5}{2}x^{-\frac{1}{2}}\right)$
22. $f'(x) = 2\left(2x^{\frac{1}{3}} + 3x^2\right)\left(\frac{2}{3}x^{-\frac{2}{3}} + 6x\right)$
23. $f'(x) = -\frac{1}{4}\left(2x^{\frac{1}{2}} + 5x\right)^{-\frac{5}{4}}\left(x^{-\frac{1}{2}} + 5\right)$
24. $f'(x) = -\frac{2}{5}\left(3x^{\frac{1}{2}} + 42x^{\frac{1}{3}}\right)^{-\frac{7}{5}}\left(\frac{3}{2}x^{-\frac{1}{2}} + 14x^{-\frac{2}{3}}\right)$
25. $f'(x) = 3(e^x + e^{-x})^2(e^x - e^{-x})$
26. $f'(x) = -e^{2x}(e^{2x} - 4e^{3x})^{-\frac{3}{2}}(1 - 6e^x)$
27. $f'(x) = \frac{2\ln(x)}{x}$
28. $f'(x) = \frac{-6}{x(\ln(2x)+\ln(x))^4}$
29. $f'(x) = 5\sin^4(x)\cos(x)$
30. $f'(x) = 3\tan^2(x)\sec^2(x)$
31. $f'(x) = \frac{2\arctan(x)}{1+x^2}$
32. $f'(x) = \frac{3\arcsin^2(x)}{\sqrt{1-x^2}}$
33. $f'(x) = 3e^{3x}$
34. $f'(x) = \cot(x)$
35. $f'(x) = \frac{1}{2\sqrt{x}(1+\sqrt{x})}$
36. $f'(x) = 3\cos(3x)$
37. $f'(x) = -\frac{1}{5}\sin\left(\frac{1}{5}x\right)$
38. $f'(x) = \sec^2(\sin(x))\cos(x)$
39. $f'(x) = -3x^2\sin(x^3)$
40. $f'(x) = \frac{3}{1+(3x)^2}$
41. $f'(x) = \frac{-e^x}{\sqrt{1-e^{2x}}}$
42. $f'(x) = x(x^2 + 3)^3(x^2 + 2)^{\frac{1}{2}}[11x^2 + 25]$
43. $f'(x) = x(x^3 + 1)^{\frac{1}{2}}(x^2 + 1)^3\left[\frac{3}{2}x(x^3 + 1)^{-1} + 8\right]$
44. $f'(x) = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$
45. $f'(x) = 2e^{2x}\tan^2(x)[\tan(x) + 3\sec^2(x)]$
46. $f'(x) = \frac{x(x^2+3)^3(11x^2+25)}{(x^2+2)^{\frac{5}{2}}}$
47. $f'(x) = \frac{e^{2x}[2\sin(3x)+3\cos(3x)]}{\sin^2(3x)}$
48. $f'(x) = 6\sin(3x)\cos(3x)$
49. $f'(x) = -4\sin(4x)e^{\cos(4x)}$
50. $f'(x) = 6\sin(2x)\cos(\cos(2x))\sin^2(\cos(2x))$
51. $f'(x) = 12\cos(3x)\tan^3(\ln(e^{\sin(3x)}))\sec^2(\ln(e^{\sin(3x)}))$

“Only he who never plays, never loses.”