The Weekly Rigor

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51 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 2)

SOLUTIONS

1. $f'(x) = 4(x^2+3)^3 2x = 8x(x^2+3)^3$.

2. $f'(x) = 5(x^3+5x^2+2)^4(3x^2+10x) = 5x(x^3+5x^2+2)^4(3x+10)$.

3. $f'(x) = \frac{3}{2}(x^2+2)^{\frac{1}{2}}2x = 3x(x^2+2)^{\frac{1}{2}} = 3x\sqrt{x^2+2}$.

4. $f'(x) = \frac{5}{3}(2x^3+3x^2+4)^{\frac{2}{3}}(6x^2+6x) = \frac{5}{3}(2x^3+3x^2+4)^{\frac{2}{3}}6x(x+1) =$
$= 10x(x+1)(2x^3+3x^2+4)^{\frac{2}{3}} = 10x(x+1)\sqrt[3]{(2x^3+3x^2+4)^2}$.

5. $f(x) = \sqrt{x^2+3} = (x^2+3)^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(x^2+3)^{-\frac{1}{2}}2x = \frac{x}{(x^2+3)^{\frac{1}{2}}} = \frac{x}{\sqrt{x^2+3}}$.

6. $f(x) = \sqrt[4]{x^3+3x^2+4} = (x^3+3x^2+4)^{\frac{1}{4}} \Rightarrow f'(x) = \frac{1}{4}(x^3+3x^2+4)^{-\frac{3}{4}}(3x^2+6x) =$
$= \frac{1}{4}(x^3+3x^2+4)^{-\frac{3}{4}}3x(x+2) = \frac{3}{4}x(x^3+3x^2+4)^{-\frac{3}{4}}(x+2) = \frac{3x(x+2)}{4(x^3+3x^2+4)^{\frac{3}{4}}} = \frac{3x(x+2)}{4\sqrt[4]{(x^3+3x^2+4)^3}}$.

7. $f(x) = \sqrt[3]{(x^2+4x)^2} = ((x^2+4x)^2)^{\frac{1}{3}} = (x^2+4x)^{\frac{2}{3}} \Rightarrow f'(x) = \frac{2}{3}(x^2+4x)^{-\frac{1}{3}}(2x+4) =$
$= \frac{2}{3}(x^2+4x)^{-\frac{1}{3}}2(x+2) = \frac{4}{3}(x^2+4x)^{-\frac{1}{3}}(x+2) = \frac{4(x+2)}{3(x^2+4x)^{\frac{1}{3}}} = \frac{4(x+2)}{3\sqrt[3]{x^2+4x}}$.

8. $f(x) = \left(\sqrt{x^3+4}\right)^3 = \left((x^3+4)^{\frac{1}{2}}\right)^3 = (x^3+4)^{\frac{3}{2}} \Rightarrow f'(x) = \frac{3}{2}(x^3+4)^{\frac{1}{2}}3x^2 = \frac{9}{2}x^2\sqrt{x^3+4}$.

9. $f'(x) = -3(x^2+2)^{-4}2x = -6x(x^2+2)^{-4} = \frac{-6x}{(x^2+2)^4}$.

10. $f'(x) = -\frac{1}{2}(2x^3+3x^2+1)^{-\frac{3}{2}}(6x^2+6x) = -\frac{1}{2}(2x^3+3x^2+1)^{-\frac{3}{2}}6x(x+1) =$
$= \frac{-3x(x+1)}{(2x^3+3x^2+1)^{\frac{3}{2}}} = \frac{-3x(x+1)}{\sqrt{(2x^3+3x^2+1)^3}}$.

11. $f(x)=\frac{1}{(2x^5+6x^2)^3}=(2x^5+6x^2)^{-3}\Rightarrow f'(x)=-3(2x^5+6x^2)^{-4}(10x^4+12x)=$
$=-3(2x^5+6x^2)^{-4}2x(5x^3+6)=\frac{-6x(5x^3+6)}{(2x^5+6x^2)^4}.$

12. $f(x)=\frac{1}{(3x^4+2x)^{\frac{5}{6}}}=(3x^4+2x)^{-\frac{5}{6}}\Rightarrow f'(x)=-\frac{5}{6}(3x^4+2x)^{-\frac{11}{6}}(12x^3+2)=$
$=-\frac{5}{6}(3x^4+2x)^{-\frac{11}{6}}2(6x^3+1)=\frac{-10(6x^3+1)}{6(3x^4+2x)^{\frac{11}{6}}}=\frac{-5(6x^3+1)}{3\sqrt[6]{(3x^4+2x)^{11}}}.$

13. $f(x)=\frac{1}{\sqrt[3]{2x-1}}=\frac{1}{(2x-1)^{\frac{1}{3}}}=(2x-1)^{-\frac{1}{3}}\Rightarrow f'(x)=-\frac{1}{3}(2x-1)^{-\frac{4}{3}}\cdot 2=\frac{-2}{3(2x-1)^{\frac{4}{3}}}=\frac{-2}{3\sqrt[3]{(2x-1)^4}}.$

14. $f(x)=\frac{1}{\sqrt{(2x^3+4x)^5}}=\frac{1}{(2x^3+4x)^{\frac{5}{2}}}=(2x^3+4x)^{-\frac{5}{2}}\Rightarrow f'(x)=-\frac{5}{2}(2x^3+4x)^{-\frac{7}{2}}(6x^2+4)=$
$=-\frac{5}{2}(2x^3+4x)^{-\frac{7}{2}}2(3x^2+2)=\frac{-5(3x^2+2)}{(2x^3+4x)^{\frac{7}{2}}}=\frac{-5(3x^2+2)}{\sqrt{(2x^3+4x)^7}}.$

15. $f'(x)=3\left(\frac{x^2+3}{x+1}\right)^2\left(\frac{2x(x+1)-(x^2+3)}{(x+1)^2}\right)=3\left(\frac{x^2+3}{x+1}\right)^2\left(\frac{x^2+2x-3}{(x+1)^2}\right)=\frac{3(x^2+3)^2(x+3)(x-1)}{(x+1)^4}.$

16. $f'(x)=4\left(\frac{x-3}{x+8}\right)^3\left(\frac{(x+8)-(x-3)}{(x+8)^2}\right)=4\left(\frac{x-3}{x+8}\right)^3\left(\frac{11}{(x+8)^2}\right)=\frac{44(x-3)^3}{(x+8)^5}.$

17. $f'(x)=3\left(x^{\frac{2}{3}}+3x^{\frac{1}{2}}\right)^2\left(\frac{2}{3}x^{-\frac{1}{3}}+\frac{3}{2}x^{-\frac{1}{2}}\right).$

18. $f(x)=\left(\sqrt{x}+4\sqrt[3]{x^2}\right)^{\frac{4}{3}}=\left(x^{\frac{1}{2}}+4x^{\frac{2}{3}}\right)^{\frac{4}{3}}\Rightarrow f'(x)=\frac{4}{3}\left(x^{\frac{1}{2}}+4x^{\frac{2}{3}}\right)^{\frac{1}{3}}\left(\frac{1}{2}x^{-\frac{1}{2}}+\frac{8}{3}x^{-\frac{1}{3}}\right).$

19. $f(x)=\sqrt[5]{1+x^{\frac{2}{3}}}=\left(1+x^{\frac{2}{3}}\right)^{\frac{1}{5}}\Rightarrow f'(x)=\frac{1}{5}\left(1+x^{\frac{2}{3}}\right)^{-\frac{4}{5}}\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{15}x^{-\frac{1}{3}}\left(1+x^{\frac{2}{3}}\right)^{-\frac{4}{5}}.$

20. $f(x)=\sqrt[4]{3\sqrt{x}+2x^{\frac{4}{3}}}=\left(3x^{\frac{1}{2}}+2x^{\frac{4}{3}}\right)^{\frac{1}{4}}\Rightarrow f'(x)=\frac{1}{4}\left(3x^{\frac{1}{2}}+2x^{\frac{4}{3}}\right)^{-\frac{3}{4}}\left(\frac{3}{2}x^{-\frac{1}{2}}+\frac{8}{3}x^{\frac{1}{3}}\right).$

21. $f'(x)=-4\left(2x^{\frac{3}{4}}+5x^{\frac{1}{2}}\right)^{-5}\left(\frac{3}{2}x^{-\frac{1}{4}}+\frac{5}{2}x^{-\frac{1}{2}}\right).$

22. $f(x)=\frac{1}{\left(2\sqrt[3]{x}+3x^2\right)^{-2}}=\left(2x^{\frac{1}{3}}+3x^2\right)^2\Rightarrow f'(x)=2\left(2x^{\frac{1}{3}}+3x^2\right)\left(\frac{2}{3}x^{-\frac{2}{3}}+6x\right).$

23. $f(x)=\frac{1}{\sqrt[4]{2\sqrt{x}+5x}}=\frac{1}{\left(2x^{\frac{1}{2}}+5x\right)^{\frac{1}{4}}}=\left(2x^{\frac{1}{2}}+5x\right)^{-\frac{1}{4}}\Rightarrow f'(x)=-\frac{1}{4}\left(2x^{\frac{1}{2}}+5x\right)^{-\frac{5}{4}}\left(x^{-\frac{1}{2}}+5\right).$