The Weekly Rigor

No. 66 "A mathematician is a machine for turning coffee into theorems." September 26, 2015

51 Problems in Calculating Derivatives Using the Chain Rule with Solutions

(Part 3)

24. $f(x) = \frac{1}{\sqrt[5]{\left(3\sqrt{x}+42x^{\frac{1}{3}}\right)^2}} = \frac{1}{\left(\left(3x^{\frac{1}{2}}+42x^{\frac{1}{3}}\right)^2\right)^{\frac{1}{5}}} = \frac{1}{\left(3x^{\frac{1}{2}}+42x^{\frac{1}{3}}\right)^{\frac{2}{5}}} = \left(3x^{\frac{1}{2}} + 42x^{\frac{1}{3}}\right)^{-\frac{2}{5}} \Rightarrow$

$\Rightarrow f'(x) = -\frac{2}{5}\left(3x^{\frac{1}{2}} + 42x^{\frac{1}{3}}\right)^{-\frac{7}{5}}\left(\frac{3}{2}x^{-\frac{1}{2}} + 14x^{-\frac{2}{3}}\right).$

25. $f'(x) = 3(e^x + e^{-x})^2(e^x - e^{-x}).$

26. $f(x) = \frac{1}{\sqrt{e^{2x}-4e^{3x}}} = \frac{1}{(e^{2x}-4e^{3x})^{\frac{1}{2}}} = (e^{2x} - 4e^{3x})^{-\frac{1}{2}} \Rightarrow f'(x) =$

$= -\frac{1}{2}(e^{2x} - 4e^{3x})^{-\frac{3}{2}}(2e^{2x} - 12e^{3x}) = -\frac{1}{2}(e^{2x} - 4e^{3x})^{-\frac{3}{2}}2e^{2x}(1 - 6e^x) =$

$= -e^{2x}(e^{2x} - 4e^{3x})^{-\frac{3}{2}}(1 - 6e^x).$

27. $f'(x) = 2\ln(x)\frac{1}{x} = \frac{2\ln(x)}{x}.$

28. $f'(x) = -3(\ln(2x) + \ln(x))^{-4}\left(\frac{2}{2x} + \frac{1}{x}\right) = -3(\ln(2x) + \ln(x))^{-4}\left(\frac{1}{x} + \frac{1}{x}\right) =$

$= -3(\ln(2x) + \ln(x))^{-4}\left(\frac{2}{x}\right) = \frac{-6}{x(\ln(2x)+\ln(x))^4}.$

29. $f'(x) = 5\sin^4(x)\cos(x).$

30. $f'(x) = 3\tan^2(x)\sec^2(x).$

31. $f'(x) = 2\arctan(x)\left(\frac{1}{1+x^2}\right) = \frac{2\arctan(x)}{1+x^2}.$

32. $f'(x) = 3\arcsin^2(x)\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{3\arcsin^2(x)}{\sqrt{1-x^2}}.$

33. $f'(x) = 3e^{3x}.$

34. $f'(x) = \frac{\cos(x)}{\sin(x)} = \cot(x).$

35. $f(x) = \ln(1 + \sqrt{x}) = \ln\left(1 + x^{\frac{1}{2}}\right) \Rightarrow f'(x) = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{1+x^{\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{2}}\left(1+x^{\frac{1}{2}}\right)} = \frac{1}{2\sqrt{x}(1+\sqrt{x})}.$

36. $f'(x) = 3\cos(3x)$.

37. $f(x) = \cos\left(\frac{x}{5}\right) = \cos\left(\frac{1}{5}x\right) \Rightarrow f'(x) = -\sin\left(\frac{1}{5}x\right)\frac{1}{5} = -\frac{1}{5}\sin\left(\frac{1}{5}x\right)$.

38. $f'(x) = \sec^{2}(\sin(x))\cos(x)$.

39. $f'(x) = -\sin(x^3)\,3x^2 = -3x^2\sin(x^3)$.

40. $f'(x) = \frac{3}{1+(3x)^2}$.

41. $f'(x) = \frac{-e^x}{\sqrt{1-(e^x)^2}} = \frac{-e^x}{\sqrt{1-e^{2x}}}$.

42. $f'(x) = 4(x^2+3)^3(2x)(x^2+2)^{\frac{3}{2}} + (x^2+3)^4\frac{3}{2}(x^2+2)^{\frac{1}{2}}(2x) =$
$= 8x(x^2+3)^3(x^2+2)^{\frac{3}{2}} + 3x(x^2+3)^4(x^2+2)^{\frac{1}{2}} = x(x^2+3)^3(x^2+2)^{\frac{1}{2}}[8(x^2+2)+3(x^2+3)] =$
$= x(x^2+3)^3(x^2+2)^{\frac{1}{2}}[8x^2+16+3x^2+9] = x(x^2+3)^3(x^2+2)^{\frac{1}{2}}[11x^2+25]$.

43. $f(x) = \sqrt{x^3+1}(x^2+1)^4 = (x^3+1)^{\frac{1}{2}}(x^2+1)^4 \Rightarrow$
$\Rightarrow f'(x) = \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)(x^2+1)^4 + (x^3+1)^{\frac{1}{2}}4(x^2+1)^3(2x) =$
$= \frac{3}{2}x^2(x^3+1)^{-\frac{1}{2}}(x^2+1)^4 + 8x(x^3+1)^{\frac{1}{2}}(x^2+1)^3 = x(x^3+1)^{\frac{1}{2}}(x^2+1)^3\left[\frac{3}{2}x(x^3+1)^{-1}+8\right]$.

44. $f'(x) = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$.

45. $f'(x) = 2e^{2x}\tan^{3}(x) + 6e^{2x}\tan^{2}(x)\sec^{2}(x) = 2e^{2x}\tan^{2}(x)\left[\tan(x) + 3\sec^{2}(x)\right]$.

46. $f'(x) = \frac{4(x^2+3)^3(2x)(x^2+2)^{\frac{3}{2}} + \frac{3}{2}(x^2+2)^{\frac{1}{2}}(2x)(x^2+3)^4}{(x^2+2)^3} = \frac{8x(x^2+3)^3(x^2+2)^{\frac{3}{2}} + 3x(x^2+2)^{\frac{1}{2}}(x^2+3)^4}{(x^2+2)^3} =$
$= \frac{x(x^2+3)^3(x^2+2)^{\frac{1}{2}}[8(x^2+2)+3(x^2+3)]}{(x^2+2)^3} = \frac{x(x^2+3)^3[8x^2+16+3x^2+9]}{(x^2+2)^{\frac{5}{2}}} = \frac{x(x^2+3)^3(11x^2+25)}{(x^2+2)^{\frac{5}{2}}}$.

47. $f'(x) = \frac{2e^{2x}\sin(3x) + 3\cos(3x)e^{2x}}{\sin^{2}(3x)} = \frac{e^{2x}[2\sin(3x)+3\cos(3x)]}{\sin^{2}(3x)}$.

48. $f'(x) = 2\sin(3x)\cos(3x)3 = 6\sin(3x)\cos(3x)$.

49. $f'(x) = e^{\cos(4x)}(-\sin(4x))4 = -4\sin(4x)e^{\cos(4x)}$.

50. $f'(x) = 3\sin^{2}(\cos(2x))\cos(\cos(2x))\sin(2x)\,2 = 6\sin(2x)\cos(\cos(2x))\sin^{2}(\cos(2x))$.

51. $f'(x) = 4\tan^{3}\left(\ln\left(e^{\sin(3x)}\right)\right)\sec^{2}\left(\ln\left(e^{\sin(3x)}\right)\right)\left(\frac{e^{\sin(3x)}\,3\cos(3x)}{e^{\sin(3x)}}\right) =$
$= 12\cos(3x)\tan^{3}\left(\ln\left(e^{\sin(3x)}\right)\right)\sec^{2}\left(\ln\left(e^{\sin(3x)}\right)\right)$.