The Weekly Rigor

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Elementary Introduction to the Exponent Rules for Positive Integer Exponents

(Part 1)

In the following, let *w*, *x*, *y*, and *z* stand for any real numbers. Let *k*, *m*, *n*, *p*, and *q* stand for any positive integers.

Definition 1: x^n means $x \cdot x \cdots x$ with *n* factors.

Remark: For example, $x^3 = xxx$. The "*x*" is called the *base*. The superscript "*n*" is called the *exponent*, which denotes the number of factors. When there is no exponent, we mean that the exponent equals 1. We say that x^n is the *nth power* of *x*. We term x^2 the *square* of *x*, and x^3 the *cube* of *x*.

Theorem 1: $x^m \cdot x^n = x^{m+n}$.

Proof: $x^m \cdot x^n \overset{\text{D1}}{=} \overbrace{(x \cdot x \cdots x)}^{m\,factors} \cdot \overbrace{(x \cdot x \cdots x)}^{n\,factors} = \overbrace{x \cdot x \cdots x}^{m+n\,factors} \overset{\text{D1}}{=} x^{m+n}$.

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Examples: $2^3 \cdot 2^4 = 2^{3+4} = 2^7$. $a^1 \cdot a^5 = a^{1+5} = a^6$.

Theorem 2: $(x^m)^n = x^{mn}$.

Proof: $(x^m)^n \overset{\text{D1}}{\triangleq} \overbrace{(x^m)\cdot(x^m)\cdots(x^m)}^{n\,factors} \overset{\text{D1}}{\triangleq} \overbrace{\overbrace{(x\cdot x\cdots x)}^{m\,factors}\cdot\overbrace{(x\cdot x\cdots x)}^{m\,factors}\cdots\overbrace{(x\cdot x\cdots x)}^{m\,factors}}^{n\,factors} =$

$= \overbrace{x\cdot x\cdots x}^{mn\,factors} \overset{\text{D1}}{\triangleq} x^{mn}$.

Examples: $(2^3)^5 = 2^{3\cdot 5} = 2^{15}$. $(a^2)^4 = a^{2\cdot 4} = a^8$.

Theorem 3: $(xy)^n = x^n y^n$.

Proof: $(xy)^n \overset{\text{D1}}{\triangleq} \overbrace{(xy)\cdot(xy)\cdots(xy)}^{n\,factors} = \overbrace{(x\cdot x\cdots x)}^{n\,factors}\cdot\overbrace{(y\cdot y\cdots y)}^{n\,factors} \overset{\text{D1}}{\triangleq} x^n y^n$.

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Examples: $(3a)^2 = 3^2 a^2 = 9a^2$. $(xy)^5 = x^5 y^5$.

Theorem 4: $(x^m y^n)^k = x^{mk} y^{nk}$.

Proof: $(x^m y^n)^k = [(x^m)(y^n)]^k \overset{\text{T3}}{\triangleq} (x^m)^k (y^n)^k \overset{\text{T2}}{\triangleq} x^{mk} y^{nk}$.

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Examples: $(2a^3b^2)^3 = 2^3 a^9 b^6 = 8a^9 b^6$. $(x^2y^3z^4)^5 = x^{10}y^{15}z^{20}$.