The Weekly Rigor

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Mal-Rules in Algebra

INTRODUCTION

Delimiting and classifying the types of mathematical errors is difficult. However, some patterns in algebra—called “mal-rules”—are very common. By the use of counterexamples, we will see that these patterns are indeed errors.

1. $$(a+b)^2 \neq a^2 + b^2.$$

Counterexample: Let $a = 1$ and $b = 1$. $(1+1)^2 = 2^2 = 4$. $1^2 + 1^2 = 1 + 1 = 2$. But $4 \neq 2$.

2. $$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}.$$

Counterexample: Let $a = 16$ and $b = 9$. $\sqrt{16+9} = \sqrt{25} = 5$. $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$. But $5 \neq 7$.

3. $$\frac{a+b}{a} \neq b.$$

Counterexample: Let $a = 2$ and $b = 4$. $\frac{2+4}{2} = \frac{6}{2} = 3$. But $3 \neq 4$.

4. $$\frac{a+b}{a} \neq 1 + b.$$

Counterexample: Let $a = 2$ and $b = 4$. $\frac{2+4}{2} = \frac{6}{2} = 3$. $1 + 4 = 5$. But $3 \neq 5$.

5. $$\frac{ac+b}{a} \neq c + b.$$

Counterexample: Let $a = 2$, $b = 2$, and $c = 3$. $\frac{2 \cdot 3 + 2}{2} = \frac{6+2}{2} = \frac{8}{2} = 4$. $3 + 2 = 5$. But $4 \neq 5$.

6. $$(-a)^2 \neq -a^2.$$

Counterexample: Let $a = 2$. $(-2)^2 = (-2)(-2) = 4$. $-2^2 = -4$. But $4 \neq -4$.

7. $$a - (b - c) \neq a - b - c.$$

Counterexample: Let $a = 3$, $b = 2$, and $c = 1$. $3 - (2 - 1) = 3 - 1 = 2$. $3 - 2 - 1 = 0$. But $2 \neq 0$.

8. $$(ab)^2 \neq ab^2.$$

Counterexample: Let $a = 2$ and $b = 3$. $(2 \cdot 3)^2 = 6^2 = 36$. $2 \cdot 3^2 = 2 \cdot 9 = 18$. But $36 \neq 18$.

9. $$a(b + c) \neq ab + c.$$

Counterexample: Let $a = 3$, $b = 2$, and $c = 1$. $3(2 + 1) = 3(3) = 9$. $3 \cdot 2 + 1 = 6 + 1 = 7$. But $9 \neq 7$.

10. $$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}.$$

Counterexample: Let $a = 2$ and $b = 2$. $\frac{1}{2+2} = \frac{1}{4}$. $\frac{1}{2} + \frac{1}{2} = 1$. But $\frac{1}{4} \neq 1$.

11. $$a(b + c)^2 \neq (ab + ac)^2.$$

Counterexample: Let $a = 3$, $b = 2$, and $c = 1$. $3(2 + 1)^2 = 3(3)^2 = 3(9) = 27$. $(3 \cdot 2 + 3 \cdot 1)^2 = (6 + 3)^2 = 9^2 = 81$. But $27 \neq 81$.

12. $$a(bc) \neq (ab)(ac).$$

Counterexample: Let $a = 3$, $b = 2$, and $c = 1$. $3(2 \cdot 1) = 3(2) = 6$. $(3 \cdot 2)(3 \cdot 1) = 6 \cdot 3 =$ $= 18$. But $6 \neq 18$.