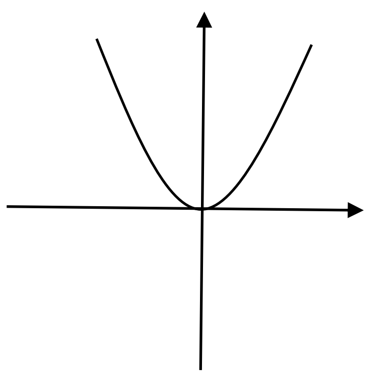
The Weekly Rigor

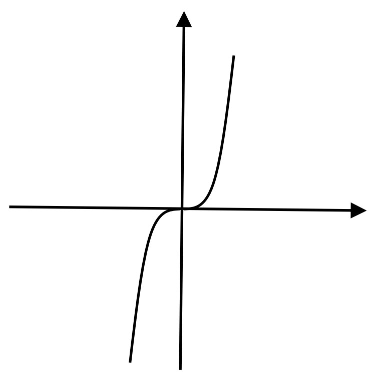
No. 92 “A mathematician is a machine for turning coffee into theorems.” March 26, 2016

An Essential Skill for Calculus Students: Graphs

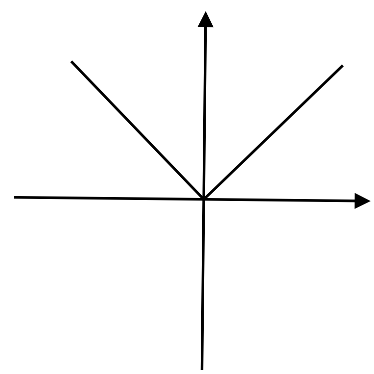
There are six basic graphs that are worth memorizing for calculus:



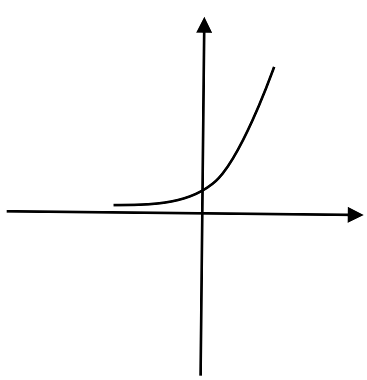
$y = x^2$



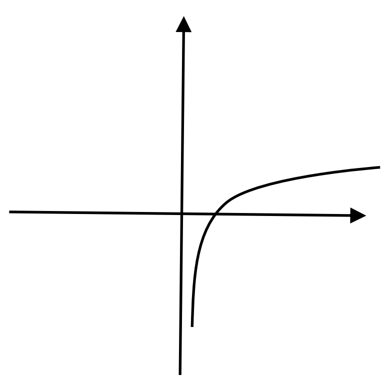
$y = x^3$



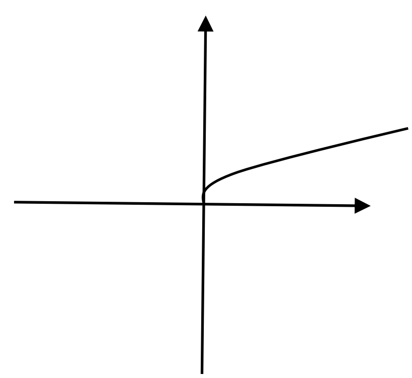
$y = |x|$



$y = e^x$



$y = \ln(x)$

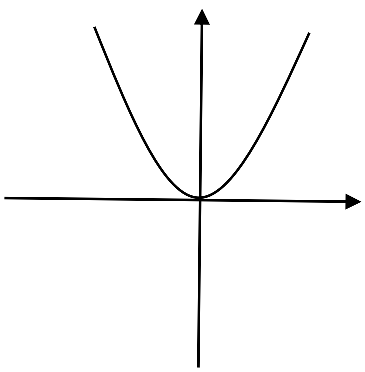


$y = \sqrt{x}$

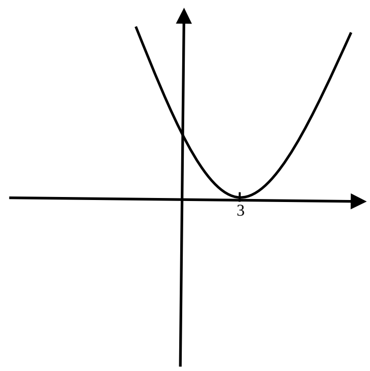
With the graphs memorized, the student can answer questions about the domain and range of the graphed functions. For example, by inspecting the graph for $y = e^x$, one can go left to right and conclude that the domain includes all x values, which in interval notation would be expressed $(-\infty, \infty)$. Furthermore, by tracking from the bottom up, one can also determine the range to be all y values greater than 0, i.e., $(0, \infty)$. A further virtue of memorizing the graph for $y = e^x$ is that one can see that as x goes off to the left in the negative direction, the function's y-values approach ever closer to 0. That is, e^x has a horizontal asymptote $y = 0$.

A second value of memorizing these graphs is that more involved functions such as $y = x^3 + 3x^2 + 5x$+10 follow the pattern of the basic graph $y = x^3$ with large values of x (whether positive or negative). The reason for this is that the leading term of the complex function—in this case x^3—tends to predominate in influence as the absolute value of x grows very large.

A final value of committing these graphs to memory is that many other graphs amount to a set of shiftings and/or reflections of the basic graphs. For example, the graph of $y = (x-3)^2$ can be thought of as the graph of the basic $y = x^2$ moved to the right three units:

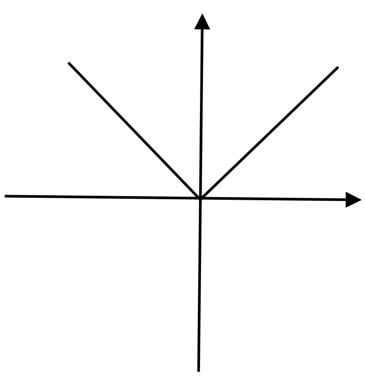


$y = x^2$

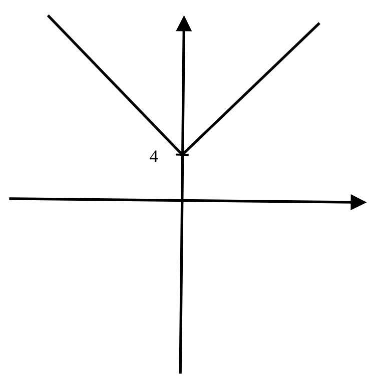


$y = (x-3)^2$

In a different case, the graph of $y = |x| + 4$ can be regarded as the basic graph of $y = |x|$ moved up four units:



$y = |x|$



$y = |x| + 4$