

# The Weekly Rigor

No. 93

“A mathematician is a machine for turning coffee into theorems.”

April 2, 2016

## An Essential Skill for Calculus Students: Linear Functions

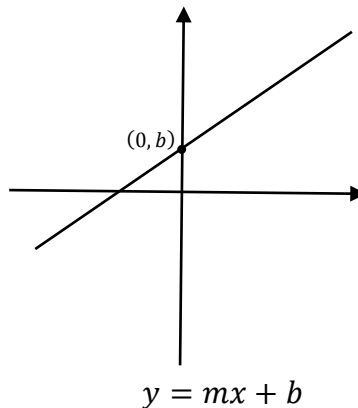
Linear functions act as a sort of foundation for calculus, both theoretically and in applications. Hence, mastering the principles of such functions is crucial for the calculus student.

To begin, a linear function is called that because its graph is a straight line. For graphing purposes, the best type of equation for a linear function is called the slope-intercept form, usually abbreviated by

$$y = mx + b,$$

Where  $m$  represents the slope and  $b$  the  $y$ -intercept.

Recall that the  $y$ -intercept always has its  $x$ -value equal to zero. In other words, the  $y$ -intercept  $b$  corresponds to the point  $(0, b)$ :



In calculus, the slope of a linear function is often called the “average rate of change.” In either case, the formula for finding the slope is “rise over run,” also known as “delta  $y$  over delta  $x$ ” or “the change in  $y$  over the change in  $x$ ”:

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

A common type of problem involving linear functions is where a point of the function is given as well as the slope, and the student is requested to write out the equation of the line. For example, it might be given that  $(3, 2)$  is a point on the line and the slope is  $m = 5$ . There are two popular ways to complete this equation, one using the point-slope form of the equation and the other going directly to the slope-intercept form.

1. Using the point-slope form. The point slope form is:

$$y_2 - y_1 = m(x_2 - x_1).$$

Hence, by substitution, we have

$$y - 2 = 5(x - 3)$$

and solving for  $y$ , we get

$$\begin{aligned}y - 2 &= 5x - 15 \\y &= 5x - 13\end{aligned}$$

2. Using the slope-intercept form. The slope-intercept form is:

$$y = mx + b.$$

Hence, by substitution, we have

$$2 = 5(3) + b$$

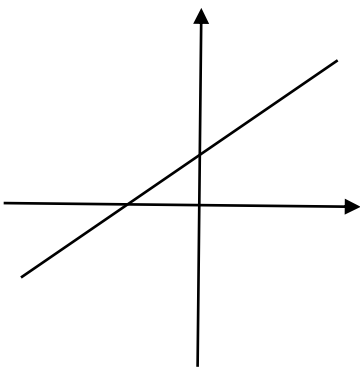
And solving for  $b$ , we get

$$\begin{aligned}2 &= 15 + b \\-13 &= b.\end{aligned}$$

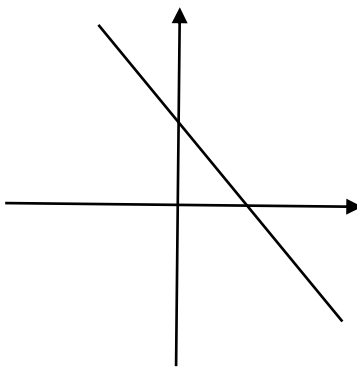
So,

$$y = 5x - 13.$$

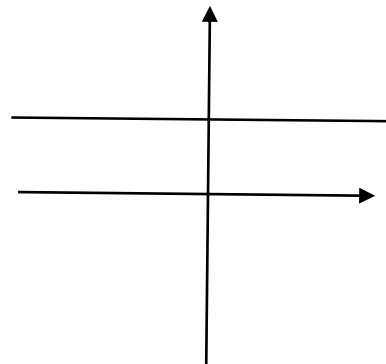
Finally, the calculus student must automatically associate a positive slope with the straight line rising from left to right, a negative slope with the straight line falling from left to right, and zero slope with a horizontal line. For example:



positive slope



negative slope



zero slope

“Only he who never plays, never loses.”