The Weekly Rigor

## An Essential Skill for Calculus Students: Plane Geometry
(Part 2)

3. The two reference triangles are the 45º-45º-90º and 30º-60º-90º triangles. These triangles are some of the most important tools in mathematics, having applications in geometry, trigonometry, calculus, and word problems.

The best way to construct the 45º-45º-90º reference triangle is to let the legs equal 1 and (by the Pythagorean Theorem) derive the hypotenuse to equal $\sqrt{2}$.
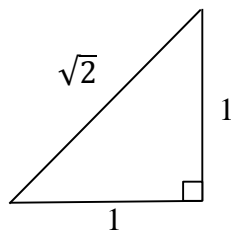

This first reference triangle is relatively easy to construct, and students almost always do so correctly. However, the second reference triangle–30º-60º-90º–is more prone to error.

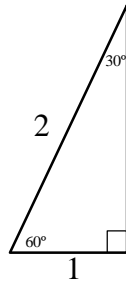To construct a correct 30º-60º-90º reference triangle, do the following steps:

1. Draw an equilateral triangle with side lengths 2. Label the fact that all the angles are 60º.
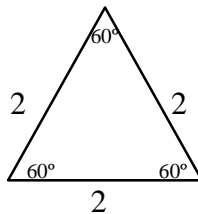
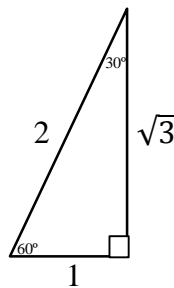2. Drop down a perpendicular bisector from the top angle to the bottom side.
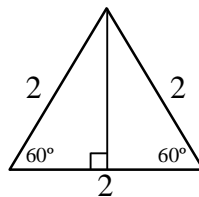
3. Focus on the left right triangle. Label the fact that the shorter leg is 1 and the top angle is 30°.

4. Use the Pythagorean Theorem to find the longer leg's length:

$$1^2 + b^2 = 2^2 \quad \Longrightarrow \quad b^2 = 4 - 1 \quad \Longrightarrow \quad b = \sqrt{3}$$

Note the fact that the hypotenuse is double the length of the shorter leg. Note further that the longer leg is $\sqrt{3}$ times the length of the shorter leg.

3. Drawing geometric figures. Drawing reasonable looking three-dimensional figures helps us to solve application problems.

Drawing a sphere comes down to drawing a circle with a horizontal oval inside:
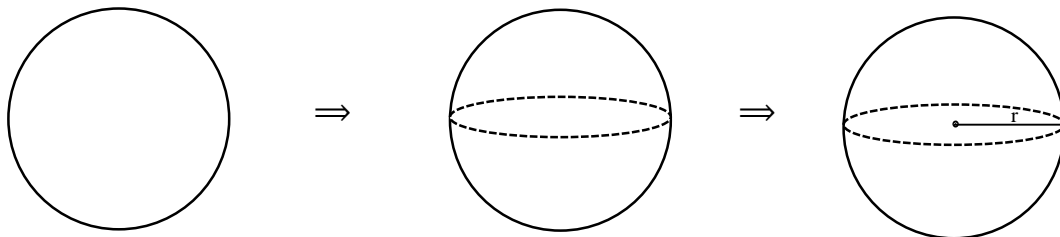
"Only he who never plays, never loses."