The Weekly Rigor

No. 98 “A mathematician is a machine for turning coffee into theorems.” May 7, 2016

An Essential Skill for Calculus Students: Trigonometry

(Part 2)

3. Needed identities. For sure a calculus student must have memorized the following trigonometric identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Furthermore, the Pythagorean identity must be automatized:

$$\boxed{\sin^2(\theta) + \cos^2(\theta) = 1}$$

There are two variations of the Pythagorean identity that are especially important in integral calculus. They can easily be derived on the fly as follows.

$\sin^2(\theta) + \cos^2(\theta) = 1$ State the Pythagorean identity.

$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$ Divide both sides by $\cos^2(\theta)$.

$\boxed{\tan^2(\theta) + 1 = \sec^2(\theta)}$ Simplify.

The second variation follows by a similar line of reasoning.

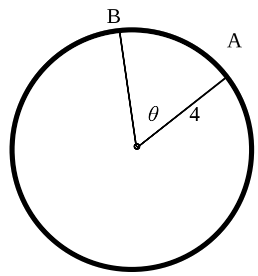
$\sin^2(\theta) + \cos^2(\theta) = 1$ State the Pythagorean identity.

$\frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$ Divide both sides by $\sin^2(\theta)$.

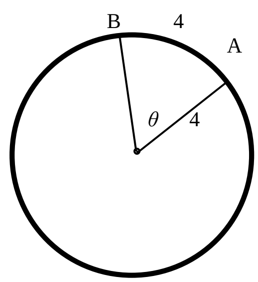
$\boxed{1 + \cot^2(\theta) = \csc^2(\theta)}$ Simplify.

4. Radian measure. When thinking of trigonometric functions as applied to circles, calculus uses only radian measure.

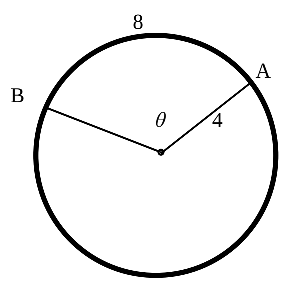
Consider a circle with central angle θ with a radius of four inches.



If the arc length of $\widehat{AB}$ also equals four inches,



then θ is said to have a radian measure of one. If instead the arc length of $\widehat{AB}$ equals eight while the radius remains at four,



then θ is said to have a radian measure of two. In other words, the radian measure begins by marking off arc lengths equal to the radius of the circle.