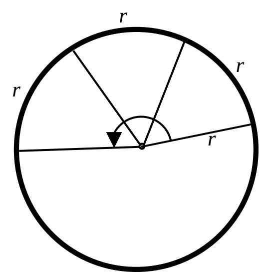
The Weekly Rigor

No. 99 “A mathematician is a machine for turning coffee into theorems.” May 14, 2016

An Essential Skill for Calculus Students: Trigonometry

(Part 3)



The central angle’s radian measure here is three.

To generalize: The radian measure of a central angle subtended by an arc is the number of radii included in the arc. Or: The radian measure of a central angle is the number of radii included in the arc intercepted by that angle. Symbolically, for arc length *s* and radius *r*, the radian measure of central angle θ is defined by

$$\theta = \frac{s}{r}.$$

For example, for an arc of length eight that subtends angle θ in a circle of radius five means that θ measures $\frac{8}{5}$ radians.

Now consider the case where the arc length involved is the entire circumference *C* of the circle, viz.,

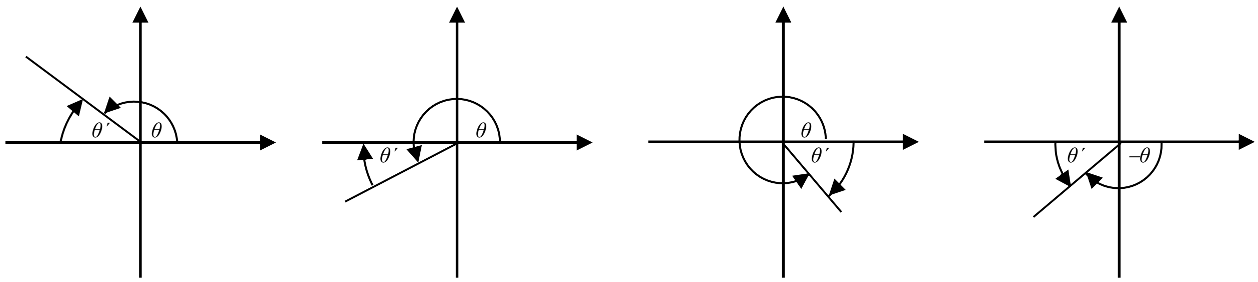
$$\theta = \frac{s}{r} = \frac{C}{r} = \frac{2\pi r}{r} = 2\pi.$$

Hence the radian measure of a central angle that involves a complete rotation equals 2π radians. So, $360° = 2\pi$ radians. (In calculus, if an angle does not have the degree mark “ ° ”, then mathematicians assume that the angle is measured in radians. Thus, we may write simply $360° = 2\pi$.)

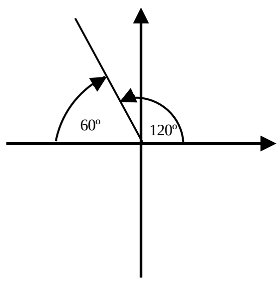
The fact that $360° = 2\pi$ implies that $180° = \pi$ allows us to state that $\frac{180°}{180} = 1° = \frac{\pi}{180}$, i.e., one degree equals $\frac{\pi}{180}$ radians. This equation gives us a way to convert from degrees to radians. For example, since $1° = \frac{\pi}{180}$, $30° = \frac{30\pi}{180} = \frac{\pi}{6}$. Furthermore, since $180° = \pi$, we can deduce that $\frac{180°}{\pi} = \frac{\pi}{\pi} = 1$, i.e., one radian equals $\frac{180}{\pi}$ degrees. This latter equation will let us convert from radians to degrees, e.g., since $\frac{180°}{\pi} = 1$, $\frac{180°}{\pi} \cdot \frac{\pi}{3} = 60° = \frac{\pi}{3}$.

5. Unit circle. Many a high school student is forced to memorize the *xy* coordinates on the unit circle. An easier way to compute these values (and more) is by the use of "reference angles."

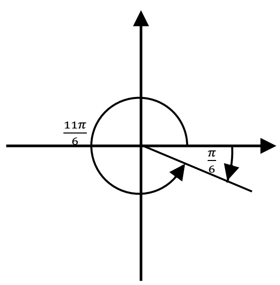
Let θ be an angle in standard position that is not a multiple of 90º. Then the reference angle for angle θ placed in standard position is the angle θ', which is the positive acute angle formed by the terminal side of θ and the horizontal axis. Examples:



(Important note: Reference angles *never* "come out" of the vertical axis.) A concrete example is the following: the reference angle of a 120º angle is 60º.



The reference angle of an angle of $\frac{11\pi}{6}$ is $\frac{\pi}{6}$:



"Only he who never plays, never loses."