The Weekly Rigor

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SAT Math Test Problem Children: Complex Numbers
(Part 2)

The College Board presents problems involving complex numbers in three formats. Here is one example of each format:

1. For $i = \sqrt{-1}$, what is the sum $(8 + 4i) + (-7 + 10i)$?

2. Which of the following complex numbers is equivalent to $\frac{5-3i}{10+4i}$? (Note: $i = \sqrt{-1}$)

A) $\frac{5}{10} - \frac{3i}{4}$

B) $\frac{5}{10} + \frac{3i}{4}$

C) $\frac{19}{58} - \frac{25i}{58}$

D) $\frac{19}{58} + \frac{25i}{58}$

3.

$$\frac{7-i}{4-3i}$$

If the expression above is rewritten in the form $a + bi$, where a and b are real numbers, what is the value of a? (Note: $i = \sqrt{-1}$)

To solve the first problem, compute the sum by adding the real parts and the imaginary parts.

$$\begin{aligned}(8 + 4i) + (-7 + 10i) &= 8 - 7 + 4i + 10i \\ &= (8 - 7) + (4 + 10)i \\ &= 1 + 14i\end{aligned}$$

To solve the second problem, start by multiplying the numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}
\frac{5-3i}{10+4i} &= \frac{5-3i}{10+4i}\cdot\frac{10-4i}{10-4i} \\
&= \frac{(5-3i)}{(10+4i)}\cdot\frac{(10-4i)}{(10-4i)} \\
&= \frac{5\cdot 10-5\cdot 4i-10\cdot 3i+3\cdot 4i^2}{10\cdot 10-10\cdot 4i+10\cdot 4i-4\cdot 4i^2} \\
&= \frac{50-20i-30i+12i^2}{100-40i+40i-16i^2} \\
&= \frac{50-50i+12i^2}{100-16i^2} \\
&= \frac{50-50i+12(-1)}{100-16(-1)} \\
&= \frac{50-50i-12}{100+16} \\
&= \frac{38-50i}{116} \\
&= \frac{38}{116}-\frac{50i}{116} \\
&= \frac{19}{58}-\frac{25i}{58}
\end{aligned}$$

Solving the third problem is very similar to the second problem. The only difference is that the final answer only specifies the real part of the complex number.

$$\begin{aligned}
\frac{7-i}{4-3i} &= \frac{(7-i)}{(4-3i)}\cdot\frac{(4+3i)}{(4+3i)} \\
&= \frac{28+17i-3i^2}{16-9i^2} \\
&= \frac{31-17i}{25} \\
&= \frac{31}{25}-\frac{17i}{25}
\end{aligned}$$

The real part being the number $\frac{31}{25}$.