

# The Weekly Rigor

No. 107

“A mathematician is a machine for turning coffee into theorems.”

July 9, 2016

## SAT Math Test Problem Children: Complex Numbers (Part 6)

### ANSWERS

1. $6 + 8i$	11. B	21. $\frac{20}{13}$
2. $12 + 8i$	12. C	22. $\frac{26}{25}$
3. $6 + 11i$	13. A	23. $\frac{28}{61}$
4. $9 + 5i$	14. C	24. $-\frac{13}{65}$
5. $4 + 15i$	15. C	25. $\frac{18}{13}$
6. $2 + 4i$	16. B	26. $-\frac{7}{25}$
7. $-1 + 13i$	17. D	27. $\frac{26}{29}$
8. $-1 + 11i$	18. D	28. $\frac{21}{17}$
9. $2 - 4i$	19. C	29. $\frac{11}{17}$
10. $1 - 8i$	20. B	30. $-\frac{27}{13}$

### SELECTED SOLUTIONS

1.

$$\begin{aligned}(2 + 3i) + (4 + 5i) &= 2 + 4 + 3i + 5i \\ &= (2 + 4) + (3 + 5)i \\ &= 6 + 8i\end{aligned}$$

11.

$$\begin{aligned}\frac{1-3i}{6+2i} &= \frac{1-3i}{6+2i} \cdot \frac{6-2i}{6-2i} \\ &= \frac{(1-3i)(6-2i)}{(6+2i)(6-2i)} \\ &= \frac{1 \cdot 6 - 1 \cdot 2i - 6 \cdot 3i + 3 \cdot 2i^2}{6 \cdot 6 - 6 \cdot 2i + 6 \cdot 2i - 2 \cdot 2i^2} \\ &= \frac{6 - 2i - 18i + 6i^2}{36 - 12i + 12i - 4i^2} \\ &= \frac{6 - 20i + 6i^2}{36 - 4i^2} \\ &= \frac{6 - 20i + 6(-1)}{36 - 4(-1)} \\ &= \frac{6 - 20i - 6}{36 + 4} \\ &= \frac{-20i}{40} \\ &= -\frac{i}{2}\end{aligned}$$

21.

$$\begin{aligned}\frac{6-i}{3-2i} &= \frac{(6-i)(3+2i)}{(3-2i)(3+2i)} \\ &= \frac{18 + 9i - 2i^2}{9 - 4i^2} \\ &= \frac{20 + 9i}{13} \\ &= \frac{20}{13} + \frac{9i}{13}\end{aligned}$$

Hence,  $a$ , the real part, is the number  $\frac{20}{13}$ .

“Only he who never plays, never loses.”