

The Weekly Rigor

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“A mathematician is a machine for turning coffee into theorems.”

July 16, 2016

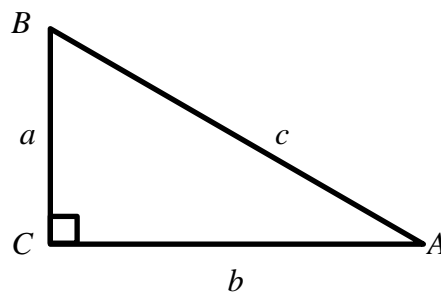
SAT Math Test Problem Children: Trigonometry (Part 1)

INTRODUCTION

The College Board has posted 280 math problems consistent with the new version of the SAT, which was launched earlier this year. These problems show up on four practice exams for the SAT and one practice exam for the PSAT. Certain categories of math questions come up repeatedly in the practice exams and are likely to challenge even the best of math students. I call these categories “problem children.” This article will address the category dealing with (right-triangle) trigonometry.

In essence, for the problems involving trigonometry, there are three definitions—united by the mnemonic “SOHCAHTOA”—and one property that the College Board calls the “complementary angle relationship.”

In a right triangle, there are three primary trigonometric functions, “sine,” “cosine,” and “tangent.” These functions are defined as follows. For a given acute angle in the triangle, the “sine” of that angle is equal to the ratio of the opposite leg divided by the hypotenuse (SOH). The “cosine” of that angle is equal to the ratio of the adjacent leg divided by the hypotenuse (CAH). The “tangent” of that angle is equal to the ratio of the opposite leg divided by the adjacent leg (TOA). For example, consider the following right triangle.



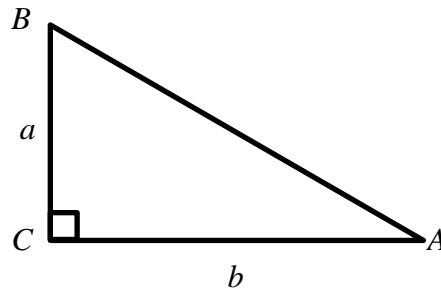
The two acute angles are labelled A and B , which are complementary, since the sum of the measures of the two angles equals 90° . By definition, $\sin(A) = \frac{a}{c}$, $\cos(A) = \frac{b}{c}$, and $\tan(A) = \frac{a}{b}$. On the other hand, $\sin(B) = \frac{b}{c}$, $\cos(B) = \frac{a}{c}$, and $\tan(B) = \frac{b}{a}$.

Now observe that in this triangle, $\sin(A) = \frac{a}{c} = \cos(B)$ and $\sin(B) = \frac{b}{c} = \cos(A)$. Hence, in general, in a right triangle the sine of one acute angle equals the cosine of the other acute angle. Furthermore, since A and B are complementary, $B^\circ = 90^\circ - A^\circ$. So, in the above

triangle, $\sin(A) = \cos(90 - A)$ and $\sin(B) = \cos(90 - B)$. The generalization of these equations to all right triangles is called the “complementary angle relationship.”

The College Board presents problems involving trigonometry in four formats. Here is one example of each format:

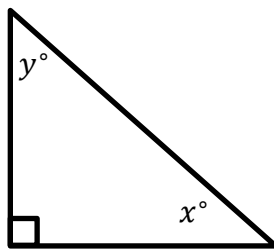
1.



Given the right triangle ABC above, which of the following is equal to $\frac{b}{a}$?

- A) $\sin A$
- B) $\sin B$
- C) $\tan A$
- D) $\tan B$

2.



In the triangle above, the sine of x° is 0.7. What is the cosine of y° ?

“Only he who never plays, never loses.”