The Weekly Rigor

No. 109

4.

"A mathematician is a machine for turning coffee into theorems."

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SAT Math Test Problem Children: Trigonometry

(Part 2)

3. In a right triangle, one angle measures x° , where $\sin x^\circ = \frac{3}{5}$. What is $\cos(90^\circ - x^\circ)$?



Note: Figures not drawn to scale.

The angles shown above are acute and $sin(a^\circ) = cos(b^\circ)$. If a = 2k - 20 and b = 8k - 11, what is the value of k?

- A) 4.5
- B) 5.5
- C) 12.1
- D) 21.1

To solve the first problem, apply "SOHCAHTOA." Since $\frac{b}{a}$ is a ratio of the right triangle's two legs, the correct answer has to involve the tangent function. Hence, the correct answer can only be C or D. However, only answer D has the correct tangent function, since relative to angle *B*, the ratio of the opposite leg divided by the adjacent leg does indeed equal $\frac{b}{a}$.

The second problem is a straightforward application of the complementary angle relationship. Angles x and y are the two complementary angles of the given right triangle. Hence, the sine of x° equals the cosine of y° . So, the cosine of y° has to equal 0.7.

The third problem is a more indirect application of the complementary angle relationship. The angle equal to $90^\circ - x^\circ$ *is* the complement of angle measuring x° . Hence, $\sin x^\circ = \cos(90^\circ - x^\circ)$. So, $\cos(90^\circ - x^\circ)$ has to equal $\frac{3}{5}$.

The fourth problem is the trickiest by far. We are given two acute angles consisting of a° and b° such that $\sin(a^{\circ}) = \cos(b^{\circ})$. By the complementary angle relationship, $\sin(a^{\circ}) = \cos(90^{\circ} - a^{\circ})$. Hence, $b^{\circ} = 90^{\circ} - a^{\circ}$, by substitution. So, since a = 2k - 20 and b = 8k - 11, it follows that 8k - 11 = 90 - (2k - 20). Thus, 8k - 11 = 90 - 2k + 20, i.e., 10k = 121. Therefore, $k = \frac{121}{10} = 12.1$, answer C.

"Only he who never plays, never loses."

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