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SAT Math Test Problem Children: Function Notation

(Part 1)

INTRODUCTION

The College Board has posted 280 math problems consistent with the new version of the SAT, which was launched earlier this year. These problems show up on four practice exams for the SAT and one practice exam for the PSAT. Certain categories of math questions come up repeatedly in the practice exams and are likely to challenge even the best of math students. I call these categories “problem children.” This article will address the category dealing with function notation.

A simple way to understand what functions do is to imagine a “rule machine” that takes in inputs and produces outputs. Let *x* represent the inputs and *y* the outputs. Now this machine produces the outputs according to a rule specified by an equation. For example, one rule machine could be:

$$x \Longrightarrow \boxed{y = 2x + 3} \Longrightarrow y$$

In words, we could say that the rule machine takes an input, multiplies it by 2, and then adds 3 to the product. Some examples of inputs becoming outputs would be:

$$1 \Longrightarrow \boxed{y = 2(1) + 3} \Longrightarrow 5$$

$$2 \Longrightarrow \boxed{y = 2(2) + 3} \Longrightarrow 7$$

$$3 \Longrightarrow \boxed{y = 2(3) + 3} \Longrightarrow 9$$

Typically the inputs and outputs are communicated as *ordered pairs* of the form (x, y). Hence, for the examples about we have $(1, 5)$, $(2, 7)$, and $(3, 9)$.

The only restriction on the function rule machine is that each input produces a *unique* output. When graphing functions, this restriction is often described as the “vertical line test.” For example, the equation $y = x^2$ is a function, since it passes the vertical line test. However, the equation $x = y^2$ is not a function, since it fails the same test.

A last issue regarding the basic concept of functions concerns the letters standing for the inputs and outputs. Let us adopt the shorthand $x \to y$ to stand in place of the rule machine diagrams we used above. With that shorthand, the student will often see $t \to P$, where t (for time) is the input and P is the output. Other popular choices are $\theta \to y$ (often used in trigonometric applications) and $t \to x$.

The ordered pair (x, y) can be written using function notation, which consists of an equation such as $f(x) = y$. Using this equation, we can identify the three parts to this notation. From left to right, the first part is the name of the function. It can be a single letter or a whole word. The second part is the pair of parentheses, which contain the input(s). Each input is called an "argument" of the function. Finally, the third part—on the right side of the equality sign—is the output corresponding to the given input. Each such output is called a "value" of the function.

$$f(x) \;=\; y$$

name argument value

The College Board presents problems involving function notation in three formats. Here is one example of each format:

1. If $f(x) = -3x + 5$, what is $f(-2x)$ equal to?

2. If $g(x) = 3x + 1$ and $f(x) = g(x) + 4$, what is $f(2)$?

3.

$$f(x) = \frac{3}{2}x + b$$

In the function above, b is a constant. If $f(6) = 8$, what is the value of $f(-2)$?

Written and published every Saturday by Richard Shedenhelm WeeklyRigor@gmail.com