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SAT Math Test Problem Children: Solving Simple Equations

(Part 1)

INTRODUCTION

The College Board has posted 280 math problems consistent with the new version of the SAT, which was launched earlier this year. These problems show up on four practice exams for the SAT and one practice exam for the PSAT. Certain categories of math questions come up repeatedly in the practice exams and are likely to challenge even the best of math students. I call these categories "problem children." This article will address the category dealing with solving simple equations.

The College Board presents problems involving solving simple equations in three formats. Here is one example of each format:

1. If $\frac{x-1}{5} = k$ and $k = 5$, what is the value of x ?

2.

$$\sqrt{x-a} = x - 5$$

If $a = 3$, what is the solution set of the equation above?

3. If $a = 6\sqrt{2}$ and $2a = \sqrt{2x}$, what is the value of x ?

There are two steps to solve the first problem: 1. Substitute the given value of k into the equation; 2. Solve for x. To wit:

$$\frac{x-1}{5} = 5$$

$$x - 1 = 25 \quad \Longrightarrow \quad x = 26$$

The second problem has three steps: 1. Substitute the given value of *a* into the equation; 2. Solve for *x*; 3. Test the solutions in the original equations to check for extraneous solutions. Following this procedure, we have:

$$\sqrt{x-3} = x - 5$$

$$\left(\sqrt{x-3}\right)^2 = (x-5)^2 \quad \Longrightarrow \quad x - 3 = x^2 - 10x + 25 \quad \Longrightarrow \quad 0 = x^2 - 11x + 28$$

$$\Longrightarrow \quad 0 = (x-7)(x-4) \quad \Longrightarrow \quad x = 7 \text{ and } x = 4$$

Testing the solutions, we have

$$\sqrt{7-3} \overset{?}{=} 7 - 5$$

$$\sqrt{4} \overset{\checkmark}{=} 2$$

However,

$$\sqrt{4-3} \overset{?}{=} 4 - 5$$

$$\sqrt{1} \neq -1$$

Hence, the solution set must omit 4.

The third question has two steps: 1. Substitute the given value of *a* into the second equation; 2. Solve for *x*. This guidance gives us:

$$2\left(6\sqrt{2}\right) = \sqrt{2x}$$

$$12\sqrt{2} = \sqrt{2}\sqrt{x} \quad \Longrightarrow \quad 12 = \sqrt{x} \quad \Longrightarrow \quad 12^2 = \left(\sqrt{x}\right)^2 \quad \Longrightarrow \quad 144 = x$$