The Weekly Rigor

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SAT Math Test Problem Children: Solving Quadratic Equations

(Part 1)

INTRODUCTION

The College Board has posted 280 math problems consistent with the new version of the SAT, which was launched earlier this year. These problems show up on four practice exams for the SAT and one practice exam for the PSAT. Certain categories of math questions come up repeatedly in the practice exams and are likely to challenge even the best of math students. I call these categories "problem children." This article will address the category dealing with quadratic equations.

The College Board presents problems involving quadratic equations in seven formats. Here is one example of each format:

1. What are the solutions to $2x^2 + 8x + 4 = 0$?

2. If $x > 0$ and $x^2 + 3x - 4 = 0$, what is the value of x?

3. What is the sum of all values of m that satisfy $3m^2 - 12m + 6 = 0$?

4.

$$2x^2 + 13x - 15 = 0$$

If r and s are two solutions of the equation above and $r > s$, what is the value of $r - s$?

5.

$$x^2 - \frac{k}{3}x = 3p$$

In the quadratic equation above, *k* and *p* are constants. What are the solutions for *x* ?

A) $x = \frac{k}{6} \pm \frac{\sqrt{k^2+2p}}{6}$

B) $x = \frac{k}{6} \pm \frac{\sqrt{k^2+108p}}{6}$

C) $x = \frac{k}{3} \pm \frac{\sqrt{k^2+2p}}{6}$

D) $x = \frac{k}{3} \pm \frac{\sqrt{k^2+108p}}{6}$

6.

$$(x+3)^2 - 16 = 0$$

What is a value of *x* that satisfies the equation above?

7. What are the solutions to the equation

$$2x^2 - 50 = 0 \text{ ?}$$

The first observation to make about these problems is that the quadratic formula either can or must be used for each of the problems. As a reminder, for a quadratic equation of the form

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

the solutions (also known as "the zeros" or "the roots") are found by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The last two problems, however, can be solved by different methods.