The Weekly Rigor

No. 120 "A mathematician is a machine for turning coffee into theorems." October 8, 2016

SAT Math Test Problem Children: Solving Quadratic Equations

(Part 2)

Now, down to the details:

1. Using the quadratic formula, with $a = 2$, $b = 8$, and $c = 4$, we have

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(4)}}{2(2)} = \frac{-8 \pm \sqrt{64 - 32}}{4} = \frac{-8 \pm \sqrt{32}}{4} = \frac{-8 \pm \sqrt{16 \cdot 2}}{4} = \frac{-8 \pm \sqrt{4^2 \cdot 2}}{4} =$$

$$= \frac{-8 \pm 4\sqrt{2}}{4} = \frac{4\left(-2 \pm \sqrt{2}\right)}{4} = -2 \pm \sqrt{2}\,.$$

Therefore, the solutions are $x = -2 + \sqrt{2}$ and $x = -2 - \sqrt{2}$.

The biggest source of error in problems of this sort have to do with cancellation. For example, if we had solutions looking like $x = \frac{-8 \pm 5\sqrt{2}}{4}$, we could *not* simplify the fraction to $-2 \pm 5\sqrt{2}$, since there is not a common factor of 4 in *both the terms of the numerator*. Also note that the College Board always reduces radicals to lowest terms, e.g., $2\sqrt{2}$, not $\sqrt{8}$.

2. Using the quadratic formula, with $a = 1$, $b = 3$, and $c = -4$, we have

$$x = \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm \sqrt{5^2}}{2} = \frac{-3 \pm 5}{2}\,.$$

Therefore, the solutions are $x = \frac{-3+5}{2} = \frac{2}{2} = 1$ and $x = \frac{-3-5}{2} = \frac{-8}{2} = -4$. However, with the constraint in the original problem that $x > 0$, the only final solution is $x = 1$.

3. Using the quadratic formula, with $a = 3$, $b = -12$, and $c = 6$, we have

$$m = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(6)}}{2(3)} = \frac{12 \pm \sqrt{144 - 72}}{6} = \frac{12 \pm \sqrt{72}}{6} = \frac{12 \pm \sqrt{36 \cdot 2}}{6} =$$

$$= \frac{12 \pm \sqrt{6^2 \cdot 2}}{6} = \frac{12 \pm 6\sqrt{2}}{6} = \frac{6\left(2 \pm \sqrt{2}\right)}{6} = 2 \pm \sqrt{2}\,.$$

Therefore, the solutions are $m = 2 + \sqrt{2}$ and $m = 2 - \sqrt{2}$. However, the problem asks for the *sum* of the two solutions. Hence, the final answer is $\left(2 + \sqrt{2}\right) + \left(2 - \sqrt{2}\right) =$
$= 2 + \sqrt{2} + 2 - \sqrt{2} = 4$.

4. Using the quadratic formula, with $a = 2$, $b = 13$, and $c = -15$, we have

$$x = \frac{-13 \pm \sqrt{(-13)^2 - 4(2)(-15)}}{2(2)} = \frac{-13 \pm \sqrt{169 + 120}}{4} = \frac{-13 \pm \sqrt{289}}{4} = \frac{-13 \pm \sqrt{17^2}}{4} =$$

$$= \frac{-13 \pm 17}{4}\,.$$

Therefore, the solutions are $x = \frac{-13+17}{4} = \frac{4}{4} = 1$ and $x = \frac{-13-17}{4} = \frac{-30}{4} = \frac{-15}{2}$. Since $1 > \frac{-15}{2}$, $r = 1$ and $s = \frac{-15}{2}$. The problem asks for the positive *difference* of the two solutions, viz., $r - s$. Hence, the final answer is $1 - \left(\frac{-15}{2}\right) = 1 + \frac{15}{2} = \frac{2}{2} + \frac{15}{2} = \frac{17}{2}$.