The Weekly Rigor

No. 121 “A mathematician is a machine for turning coffee into theorems.” October 15, 2016

SAT Math Test Problem Children: Solving Quadratic Equations
(Part 3)

5. Before using the quadratic formula, we need to arrange the terms of the equation in the standard order, in order to correctly identify the constants a, b, and c. Hence, we have

$$x^2 - \frac{k}{3}x - 3p = 0\,.$$

We will make the problem easier to solve if we eliminate the fraction in the linear term. We can accomplish this by multiplying each term by 3. So, we will have

$$3x^2 - kx - 9p = 0\,.$$

Now, using the quadratic formula, with $a = 3$, $b = -k$, and $c = -9p$, we have

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(3)(-9p)}}{2(3)} = \frac{k \pm \sqrt{k^2 + 108p}}{6} = \frac{k}{6} \pm \frac{\sqrt{k^2 + 108p}}{6}\,.$$

Therefore, the answer is option B.

6. This problem can be solved using the quadratic formula, if we first expand the first term and then collect like terms, viz.,

$$(x+3)^2 - 16 = 0 \quad \Rightarrow \quad x^2 + 6x + 9 - 16 = 0 \quad \Rightarrow \quad x^2 + 6x - 7 = 0\,.$$

Using the quadratic formula, with $a = 1$, $b = 6$, and $c = -7$, we have

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-7)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 28}}{2} = \frac{-6 \pm \sqrt{64}}{2} = \frac{-6 \pm 8}{2} = \frac{2(-3 \pm 4)}{2} =$$

$$= -3 \pm 4\,.$$

Therefore, the solutions are $x = -3 + 4 = 1$ and $x = -3 - 4 = -7$.

An alternative way to solve this problem—not using the quadratic formula—goes as follows.

$$(x+3)^2 - 16 = 0 \quad \Longrightarrow \quad (x+3)^2 = 16 \quad \Longrightarrow \quad \sqrt{(x+3)^2} = \pm\sqrt{16} \quad \Longrightarrow \quad x+3 = \pm 4$$

$$\Longrightarrow \quad x = -3 \pm 4\,.$$

Therefore, as before, the solutions are $x = -3 + 4 = 1$ and $x = -3 - 4 = -7$.

7. This problem can be solved using the quadratic formula, just by letting $b = 0$. Hence, using the quadratic formula, with $a = 2$, $b = 0$, and $c = -50$, we have

$$x = \frac{-0 \pm \sqrt{0^2 - 4(2)(-50)}}{2(2)} = \frac{\pm\sqrt{400}}{4} = \frac{\pm 20}{4} = \pm 5.$$

Therefore, the solutions are $x = 5$ and $x = -5$.

An alternative way to solve this problem—not using the quadratic formula—goes as follows.

$$2x^2 - 50 = 0 \quad \Longrightarrow \quad 2x^2 = 50 \quad \Longrightarrow \quad x^2 = 25 \quad \Longrightarrow \quad \sqrt{x^2} = \pm 5 \quad \Longrightarrow \quad x = \pm 5.$$

The advantage to doing problems like 6 and 7 using the quadratic formula is that we will have a single method to solve *all* questions requiring us to solve quadratic equations. In addition, many students make the mistake of forgetting the "$\pm$" in the square-rooting step of the alternative method, e.g., $(x+3)^2 = 16 \quad \Longrightarrow \quad \sqrt{(x+3)^2} = \pm\sqrt{16}$.