The Weekly Rigor

No. 124 "A mathematician is a machine for turning coffee into theorems." November 5, 2016

SAT Math Test Problem Children: Solving Quadratic Equations
(Part 6)

SELECTED SOLUTIONS

1. Using the quadratic formula, with $a = 2$, $b = 8$, and $c = 2$, we have

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(2)}}{2(2)} = \frac{-8 \pm \sqrt{64 - 16}}{4} = \frac{-8 \pm \sqrt{48}}{4} = \frac{-8 \pm \sqrt{16 \cdot 3}}{4} = \frac{-8 \pm \sqrt{4^2 \cdot 3}}{4} =$$

$$= \frac{-8 \pm 4\sqrt{3}}{4} = \frac{4\left(-2 \pm \sqrt{3}\right)}{4} = -2 \pm \sqrt{3}\,.$$

Therefore, the solutions are $x = -2 + \sqrt{3}$ and $x = -2 - \sqrt{3}$.

Alternative solution:

$$2x^2 + 8x + 2 = 0 \quad \Longrightarrow \quad 2(x^2 + 4x + 1) = 0 \quad \Longrightarrow \quad x^2 + 4x + 1 = 0.$$

Using the quadratic formula, with $a = 1$, $b = 4$, and $c = 1$, we have

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 4}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm \sqrt{4 \cdot 3}}{2} = \frac{-4 \pm \sqrt{2^2 \cdot 3}}{2} =$$

$$= \frac{-4 \pm 2\sqrt{3}}{2} = \frac{2\left(-2 \pm \sqrt{3}\right)}{2} = -2 \pm \sqrt{3}\,.$$

Therefore, as before, the solutions are $x = -2 + \sqrt{3}$ and $x = -2 - \sqrt{3}$.

4. Using the quadratic formula, with $a = 5$, $b = 4$, and $c = -1$, we have

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(5)(-1)}}{2(5)} = \frac{-4 \pm \sqrt{16 + 20}}{10} = \frac{-4 \pm \sqrt{36}}{10} = \frac{-4 \pm \sqrt{6^2}}{10} = \frac{-4 \pm 6}{10}.$$

Therefore, the solutions are $x = \frac{-4+6}{10} = \frac{2}{10} = \frac{1}{5}$ and $x = \frac{-4-6}{10} = \frac{-10}{10} = -1$. However, with the constraint in the original problem that $x > 0$, the only final solution is $x = \frac{1}{5}$.

7. Using the quadratic formula, with $a = 3$, $b = -12$, and $c = 3$, we have

$$m = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(3)}}{2(3)} = \frac{12 \pm \sqrt{144 - 36}}{6} = \frac{12 \pm \sqrt{108}}{6} = \frac{12 \pm \sqrt{36 \cdot 3}}{6} =$$

$$= \frac{12 \pm \sqrt{6^2 \cdot 3}}{6} = \frac{12 \pm 6\sqrt{3}}{6} = \frac{6\left(2 \pm \sqrt{3}\right)}{6} = 2 \pm \sqrt{3}\,.$$

Therefore, the solutions are $m = 2 + \sqrt{3}$ and $m = 2 - \sqrt{3}$. However, the problem asks for the *sum* of the two solutions. Hence, the final answer is $\left(2 + \sqrt{3}\right) + \left(2 - \sqrt{3}\right) =$
$= 2 + \sqrt{3} + 2 - \sqrt{3} = 4$.

Alternative solution:

$$3m^2 - 12m + 3 = 0 \quad \Longrightarrow \quad 3(m^2 - 4m + 1) = 0 \quad \Longrightarrow \quad m^2 - 4m + 1 = 0.$$

Using the quadratic formula, with $a = 1$, $b = -4$, and $c = 1$, we have

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm \sqrt{4 \cdot 3}}{2} = \frac{4 \pm \sqrt{2^2 \cdot 3}}{2} =$$

$$= \frac{4 \pm 2\sqrt{3}}{2} = \frac{2\left(2 \pm \sqrt{3}\right)}{2} = 2 \pm \sqrt{3}\,.$$

Therefore, as before, the initial solutions are $x = 2 + \sqrt{3}$ and $x = 2 - \sqrt{3}$.