The Weekly Rigor

No. 125 "A mathematician is a machine for turning coffee into theorems." November 12, 2016

SAT Math Test Problem Children: Solving Quadratic Equations

(Part 7)

10. Using the quadratic formula, with $a = 3$, $b = 6$, and $c = -9$, we have

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(-9)}}{2(3)} = \frac{-6 \pm \sqrt{36 + 108}}{6} = \frac{-6 \pm \sqrt{144}}{6} = \frac{-6 \pm \sqrt{12^2}}{6} =$$

$$= \frac{-6 \pm 12}{6}.$$

Therefore, the solutions are $x = \frac{-6+12}{6} = \frac{6}{6} = 1$ and $x = \frac{-6-12}{6} = \frac{-18}{6} = -3$. However, the problem asks for the positive *difference* of the two solutions. Hence, the final answer is

$1 - (-3) = 1 + 3 = 4$.

Alternative solution:

$3x^2 + 6x - 9 = 0 \quad \Longrightarrow \quad 3(x^2 + 2x - 3) = 0 \quad \Longrightarrow \quad x^2 + 2x - 3 = 0.$

Using the quadratic formula, with $a = 1$, $b = 2$, and $c = -3$, we have

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm \sqrt{4^2}}{2} =$$

$$= \frac{-2 \pm 4}{2} = \frac{2(-1 \pm 2)}{2} = -1 \pm 2 .$$

Therefore, as before, the initial solutions are $x = -1 + 2 = 1$ and $x = -1 - 2 = -3$.

13. Before using the quadratic formula, we need to arrange the terms of the equation in the standard order, in order to correctly identify the constants a, b, and c. Hence, we have

$$x^2 - \frac{k}{2}x - 2p = 0\,.$$

We will make the problem easier to solve if we eliminate the fraction in the linear term. We can accomplish this by multiplying each term by 2. So, we will have

$$2x^2 - kx - 4p = 0\,.$$

Now, using the quadratic formula, with $a = 2$, $b = -k$, and $c = -4p$, we have

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(2)(-4p)}}{2(2)} = \frac{k \pm \sqrt{k^2 + 32p}}{4} = \frac{k}{4} \pm \frac{\sqrt{k^2 + 32p}}{4}\,.$$

Therefore, the answer is option D.

14. Before using the quadratic formula, we need to arrange the terms of the equation in the standard order, in order to correctly identify the constants a, b, and c. Hence, we have

$$x^2 - \frac{k}{4}x - 4p = 0\,.$$

We will make the problem easier to solve if we eliminate the fraction in the linear term. We can accomplish this by multiplying each term by 4. So, we will have

$$4x^2 - kx - 16p = 0\,.$$

Now, using the quadratic formula, with $a = 4$, $b = -k$, and $c = -16p$, we have

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(4)(-16p)}}{2(4)} = \frac{k \pm \sqrt{k^2 + 256p}}{8} = \frac{k}{8} \pm \frac{\sqrt{k^2 + 256p}}{8}\,.$$

Therefore, the answer is option C.