The Weekly Rigor

No. 126 "A mathematician is a machine for turning coffee into theorems." November 19, 2016

SAT Math Test Problem Children: Solving Quadratic Equations

(Part 8)

15. Before using the quadratic formula, we need to arrange the terms of the equation in the standard order, in order to correctly identify the constants *a*, *b*, and *c*. Hence, we have

$$x^2 - \frac{k}{2}x - 3p = 0\,.$$

We will make the problem easier to solve if we eliminate the fraction in the linear term. We can accomplish this by multiplying each term by 2. So, we will have

$$2x^2 - kx - 6p = 0\,.$$

Now, using the quadratic formula, with $a = 2$, $b = -k$, and $c = -6p$, we have

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(2)(-6p)}}{2(2)} = \frac{k \pm \sqrt{k^2 + 48p}}{4} = \frac{k}{4} \pm \frac{\sqrt{k^2 + 48p}}{4}\,.$$

Therefore, the answer is option D.

16. This problem can be solved using the quadratic formula, if we first expand the first term and then collect like terms, viz.,

$$(x+4)^2 - 9 = 0 \quad \Longrightarrow \quad x^2 + 8x + 16 - 9 = 0 \quad \Longrightarrow \quad x^2 + 8x + 7 = 0\,.$$

Using the quadratic formula, with $a = 1$, $b = 8$, and $c = 7$, we have

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(7)}}{2(1)} = \frac{-8 \pm \sqrt{64 - 28}}{2} = \frac{-8 \pm \sqrt{36}}{2} = \frac{-8 \pm 6}{2} = \frac{2(-4 \pm 3)}{2} =$$

$$= -4 \pm 3\,.$$

Therefore, the solutions are $x = -4 + 3 = -1$ and $x = -4 - 3 = -7$.

Alternative solution:

$$(x+4)^2 - 9 = 0 \quad \Longrightarrow \quad (x+4)^2 = 9 \quad \Longrightarrow \quad \sqrt{(x+4)^2} = \pm\sqrt{9} \quad \Rightarrow \quad x + 4 = \pm 3$$

$\Rightarrow \quad x = -4 \pm 3$.

Therefore, as before, the solutions are $x = -4 + 3 = -1$ and $x = -4 - 3 = -7$.

19. This problem can be solved using the quadratic formula, just by letting $b = 0$. Hence, using the quadratic formula, with $a = 2$, $b = 0$, and $c = -72$, we have

$$x = \frac{-0 \pm \sqrt{0^2 - 4(2)(-72)}}{2(2)} = \frac{\pm\sqrt{576}}{4} = \frac{\pm 24}{4} = \pm 6.$$

Therefore, the solutions are $x = 6$ and $x = -6$.

We can make the above approach to solving the problem easier if we first factor out the greatest common factor:

$$2x^2 - 72 = 0 \quad \Longrightarrow \quad 2(x^2 - 36) = 0 \quad \Longrightarrow \quad x^2 - 36 = 0.$$

Now we can use the quadratic formula with $a = 1$, $b = 0$, and $c = -36$. Hence, we have

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(-36)}}{2(1)} = \frac{\pm\sqrt{144}}{2} = \frac{\pm 12}{2} = \pm 6.$$

Alternative solution:

$$2x^2 - 72 = 0 \quad \Longrightarrow \quad 2x^2 = 72 \quad \Longrightarrow \quad x^2 = 36 \quad \Longrightarrow \quad \sqrt{x^2} = \pm 6 \quad \Rightarrow \quad x = \pm 6.$$

Therefore, as before, the solutions are $x = 6$ and $x = -6$.