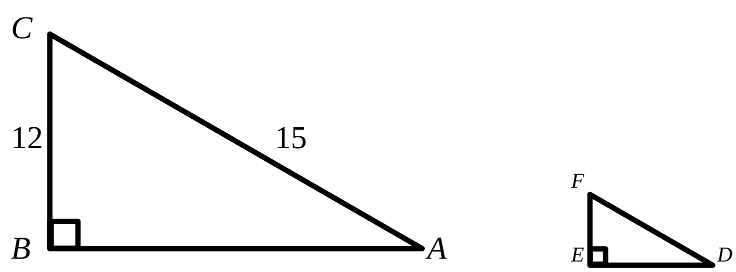
The Weekly Rigor

No. 128 “A mathematician is a machine for turning coffee into theorems.” December 3, 2016

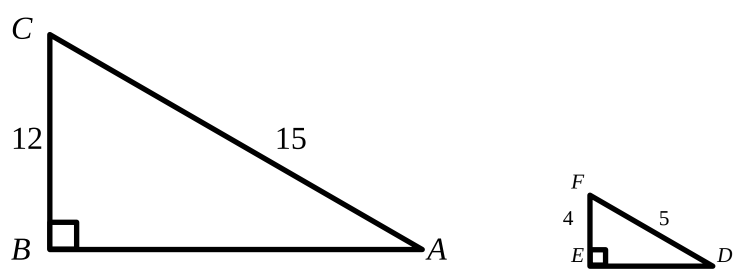
SAT Math Test Problem Children: Geometry

(Part 2)

To solve the first problem, begin by drawing representative triangles.



Since the sides of triangle DEF are $\frac{1}{3}$ the length of the corresponding sides of triangle ABC, we can fill in the lengths of EF and DF:



The value of sin F is equal the ratio $\frac{DE}{DF}$. Hence, we need to use the Pythagorean Theorem to find the length of DE. $DE^2 + 4^2 = 5^2$. So, $DE^2 = 25 - 16 = 9$. Thus, $DE = 3$. Therefore, $\sin F = \frac{3}{5}$.

To solve the second problem, note that $\overline{AE} \parallel \overline{CD}$. Hence, $\angle C \cong \angle E$ and $\angle D \cong \angle A$. Furthermore, vertical angles $\angle CBD$ and $\angle EBA$ are also congruent. So, $\triangle CBD$ is similar to $\triangle EBA$. Thus, $\frac{CB}{6} = \frac{8}{12}$. Hence, $CB = \frac{6 \cdot 8}{12} = \frac{48}{12} = 4$. Therefore, $CE = 4 + 8 = 12$.

To solve the third problem, first note that since *LM* and *MN* are tangent to the circle at points *L* and *N*, respectively, both $\angle MLO$ and $\angle MNO$ are right angles. Hence, $\angle MLO$, $\angle LMN$ and $\angle MNO$ add up to $90° + 60° + 90° = 240°$. However, all the angles of quadrilateral *OLMN* sum up to a total of $360°$. So, $\angle LON = 360° - 240° = 120°$. Thus, since 120 is one third of 360, the length of minor arc $\widehat{LN}$ is one third of the circle's circumference. Therefore, the arc's length is $\frac{93}{3} = 31$.

Solving the fourth problem, begin by noting that vertical angles *y* and *u* are congruent. Hence, $x + y = y + w$. So, $x = w$. But vertical angles *w* and *z* are congruent. Consequently, $x = z$ (option I). Given the assumptions of this problem, equal angles *y* and *u* could both be $80°$ and both *x* and *w* could be, say, $40°$. In that case, $y \neq w$ and $x \neq u$. Therefore, only option I *must* be true—choice B.