The Weekly Rigor

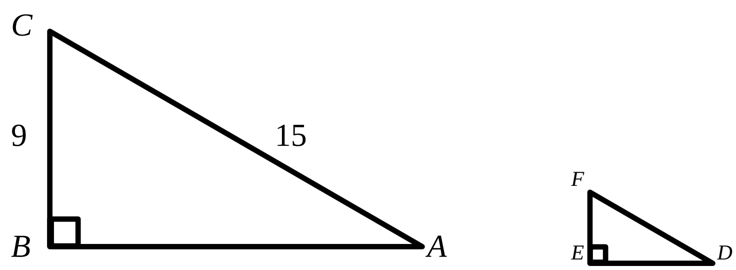
No. 132 “A mathematician is a machine for turning coffee into theorems.” December 31, 2016

SAT Math Test Problem Children: Geometry

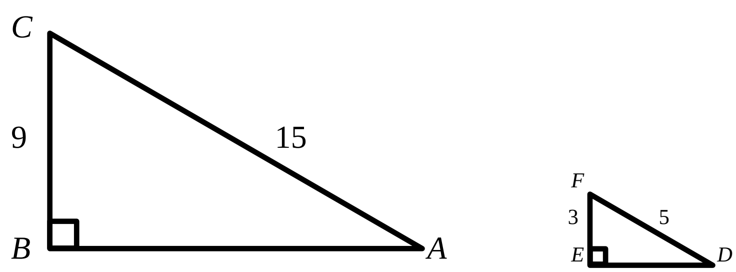
(Part 6)

SELECTED SOLUTIONS

1. To solve this problem, begin by drawing representative triangles.



Since the sides of triangle *DEF* are $\frac{1}{3}$ the length of the corresponding sides of triangle *ABC*, we can fill in the lengths of *EF* and *DF*:



The value of sin *F* is equal the ratio $\frac{DE}{DF}$. Hence, we need to use the Pythagorean Theorem to find the length of *DE*. $DE^2 + 3^2 = 5^2$. So, $DE^2 = 25 - 9 = 16$. Thus, $DE = 4$. Therefore, $\sin F = \frac{4}{5}$.

4. To solve this, note that $\overline{AE} \parallel \overline{CD}$. Hence, $\angle C \cong \angle E$ and $\angle D \cong \angle A$. Furthermore, vertical angles $\angle CBD$ and $\angle EBA$ are also congruent. So, $\triangle CBD$ is similar to $\triangle EBA$. Thus, $\frac{CB}{4} = \frac{6}{8}$. Hence, $CB = \frac{4 \cdot 6}{8} = \frac{24}{8} = 3$. Therefore, $CE = 3 + 6 = 9$.

7. To solve this problem, first note that since *LM* and *MN* are tangent to the circle at points *L* and *N*, respectively, both $\angle MLO$ and $\angle MNO$ are right angles. Hence, $\angle MLO$, $\angle LMN$ and $\angle MNO$ add up to $90° + 60° + 90° = 240°$. However, all the angles of quadrilateral *OLMN* sum up to a total of $360°$. So, $\angle LON = 360° - 240° = 120°$. Thus, since 120 is one third of 360, the length of minor arc $\widehat{LN}$ is one third of the circle's circumference. Therefore, the arc's length is $\frac{99}{3} = 33$.

10. In solving this problem, begin by noting that vertical angles *y* and *u* are congruent (option II). Hence, $x + y = y + w$. So, $x = w$. But vertical angles *x* and *t* are congruent. Consequently, $w = t$ (option I). Given the assumptions of this problem, equal angles *y* and *u* could both be $80°$ and both *x* and *w* could be, say, $40°$. In that case, $w \neq u$. Therefore, only options I and II *must* be true—choice A.