The Weekly Rigor

No. 135 "A mathematician is a machine for turning coffee into theorems." January 21, 2017

SAT Math Test Problem Children: Systems of Linear Equations

(Part 3)

3. First note that the problem states $b = c - \frac{1}{2}$. Hence, by substitution into the first equation,

$$3x + \left(c - \frac{1}{2}\right) = 5x - 7.$$

So,

$$c = 2x - 7 + \frac{1}{2} = 2x - \frac{14}{2} + \frac{1}{2} = 2x - \frac{13}{2}.$$

Thus, by substitution into the second equation,

$$3y + \left(2x - \frac{13}{2}\right) = 5y - 7.$$

Hence,

$$2x = 2y - 7 + \frac{13}{2} = 2y - \frac{14}{2} + \frac{13}{2} = 2y - \frac{1}{2}.$$

So,

$$x = y - \frac{1}{4},$$

which is option A.

4. The condition that the system has infinitely many solutions implies that the two equations are really the same! Multiplying the first equation by 5, the system becomes

$$\begin{aligned} 5ax + 5by &= 60 \\ 2x + 8y &= 60 \end{aligned}$$

Hence,

$$5a = 2 \text{ and } 5b = 8.$$

So,

$$a = \tfrac{2}{5} \text{ and } b = \tfrac{8}{5}.$$

Therefore,

$$\frac{a}{b} = \frac{\left(\frac{2}{5}\right)}{\left(\frac{8}{5}\right)} = \frac{2}{5} \cdot \frac{5}{8} = \frac{2}{8} = \frac{1}{4}.$$

5. The condition that the system has no solution implies that the two equations represent two distinct but parallel lines. Hence, the slopes of the lines are the same. Converting the equations into slope-intercept form, we have

$$kx - 3y = 4 \quad \Longrightarrow \quad kx - 4 = 3y \quad \Longrightarrow \quad \frac{k}{3}x - \frac{4}{3} = y \quad \Longrightarrow \quad y = \frac{k}{3}x - \frac{4}{3}$$

and

$$4x - 5y = 7 \quad \Longrightarrow \quad 4x - 7 = 5y \quad \Longrightarrow \quad \frac{4}{5}x - \frac{7}{5} = y \quad \Longrightarrow \quad y = \frac{4}{5}x - \frac{7}{5}.$$

(Note that the equations have distinct *y*-intercepts.) So,

$$\frac{k}{3} = \frac{4}{5}.$$

Thus,

$$k = \frac{12}{5}$$

is the value of *k* that will render the system of equations to have no solution.

6. Since we want the equation that represents a line parallel to the equation

$$y = -3x + 4,$$

we need to find the equation that represents a line with the same slope, viz., $m = -3$. Converting equation A into slope-intercept form, we have

$$6x + 2y = 15 \quad \Longrightarrow \quad 2y = -6x + 15 \quad \Longrightarrow \quad y = -3x + \frac{15}{2}.$$

Hence, equation A has the slope $m = -3$. So, option A is the correct answer.

7. Converting the second equation into slope-intercept form, we have

$$4x - 4y = 12 \quad \Longrightarrow \quad 4x - 12 = 4y \quad \Longrightarrow \quad x - 3 = y \quad \Longrightarrow \quad y = x - 3.$$

Hence, the slope of both lines is $m = 1$. However, the *y*-intercepts of the two lines are not the same. Therefore, we have two distinct but parallel lines, viz., option A.