

The Weekly Rigor

No. 139

“A mathematician is a machine for turning coffee into theorems.”

February 18, 2017

SAT Math Test Problem Children: Systems of Linear Equations

(Part 7)

SELECTED SOLUTIONS

1. Using the “elimination method,” we have

$$\begin{array}{r} x + y = 2 \\ 5x - y = 22 \\ \hline 6x = 24 \end{array}$$

Hence,

$$\frac{6x}{6} = \frac{24}{6}.$$

So,

$$x = 4.$$

Thus,

$$4 + y = 2 \implies y = 2 - 4 = -2.$$

Alternative solution:

Using the “substitution method,” we could begin as follows:

$$y = 2 - x.$$

Hence,

$$5x - (2 - x) = 22,$$

by substitution. So,

$$5x - 2 + x = 22 \implies 6x = 24 \implies x = 4.$$

Thus,

$$y = 2 - 4 = -2.$$

11. Using the “elimination method,” we have

$$\begin{aligned}(-3)(3x - 4y) &= -11(-3) \\(4)(4x - 3y) &= 4(4)\end{aligned}$$

Hence,

$$\begin{aligned}-9x + 12y &= 33 \\ \underline{16x - 12y} &= \underline{16} \\ 7x &= 49\end{aligned}$$

So,

$$\frac{7x}{7} = \frac{49}{7}.$$

Thus,

$$x = 7.$$

Hence,

$$3(7) - 4y = -11 \Rightarrow 21 - 4y = -11 \Rightarrow -4y = -32.$$

So,

$$y = 8.$$

Therefore,

$$x - y = 7 - 8 = -1,$$

which is option C.

13. First note that the problem states $b = c - \frac{1}{2}$. Hence, by substitution into the first equation,

$$2x + \left(c - \frac{1}{2}\right) = 4x - 6.$$

So,

$$c = 2x - 6 + \frac{1}{2} = 2x - \frac{12}{2} + \frac{1}{2} = 2x - \frac{11}{2}.$$

Thus, by substitution into the second equation,

$$2y + \left(2x - \frac{11}{2}\right) = 4y - 6.$$

Hence,

$$2x = 2y - 6 + \frac{11}{2} = 2y - \frac{12}{2} + \frac{11}{2} = 2y - \frac{1}{2}.$$

So,

$$x = y - \frac{1}{4},$$

which is option D.

“Only he who never plays, never loses.”