The Weekly Rigor

No. 140 "A mathematician is a machine for turning coffee into theorems." February 25, 2017

SAT Math Test Problem Children: Systems of Linear Equations

(Part 8)

15. The condition that the system has infinitely many solutions implies that the two equations are really the same equation. Multiplying the first equation by 6, the system becomes

$$\begin{aligned} 6ax + 6by &= 54 \\ 3x + 4y &= 54 \end{aligned}$$

Hence,

$$6a = 3 \text{ and } 6b = 4.$$

So,

$$a = \tfrac{3}{6} \text{ and } b = \tfrac{4}{6}.$$

Therefore,

$$\frac{a}{b} = \frac{\left(\frac{3}{6}\right)}{\left(\frac{4}{6}\right)} = \frac{3}{6} \cdot \frac{6}{4} = \frac{3}{4}.$$

17. The condition that the system has no solution implies that the two equations represent two distinct but parallel lines. Hence, the slopes of the lines are the same. Converting the equations into slope-intercept form, we have

$$kx - 2y = 5 \implies kx - 5 = 2y \implies \frac{k}{2}x - \frac{5}{2} = y \implies y = \frac{k}{2}x - \frac{5}{2}$$

and

$$3x - 4y = 8 \implies 3x - 8 = 4y \implies \frac{3}{4}x - \frac{8}{4} = y \implies y = \frac{3}{4}x - 2\,.$$

(Note that the equations have distinct *y*-intercepts.) So,

$$\frac{k}{2} = \frac{3}{4}.$$

Thus,

$$k = \frac{6}{4} = \frac{3}{2}$$

is the value of *k* that will render the system of equations to have no solution.

19. Since we want the equation that represents a line parallel to the equation

$$y = -4x + 4,$$

we need to find the equation that represents a line with the same slope, viz., $m = -4$. Converting equation C into slope-intercept form, we have

$$8x + 2y = 6 \quad \Longrightarrow \quad 2y = -8x + 6 \quad \Longrightarrow \quad y = -4x + 3.$$

Hence, equation C has the slope $m = -4$. So, option C is the correct answer.

21. Converting the second equation into slope-intercept form, we have

$$2y - 2x = 6 \quad \Longrightarrow \quad 2y = 2x + 6 \quad \Longrightarrow \quad y = x + 3.$$

Hence, the slope of both lines is $m = 1$. However, the *y*-intercepts of the two lines are not the same. Therefore, we have two distinct but parallel lines, viz., option A.

22. Converting the second equation into slope-intercept form, we have

$$3x - 4y = 10 \quad \Longrightarrow \quad -4y = -3x + 10 \quad \Longrightarrow \quad y = \frac{3}{4}x - \frac{5}{2}.$$

Hence, the slopes of both lines are neither the same nor negative reciprocals of one another. So, options A, B, and C cannot be true. As extra confirmation that option D is correct, note that by substituting $x = -30$ and $y = -25$ we get the true equations

$$-25 = -30 + 5$$
$$3(-30) - 4(-25) = 10.$$

23. Converting the second equation into slope-intercept form, we have

$$x + 2y = 16 \quad \Longrightarrow \quad 2y = -x + 16 \quad \Longrightarrow \quad y = -\frac{1}{2}x + 8.$$

Hence, the slopes of both lines are negative reciprocals of one another. So, option C is correct.