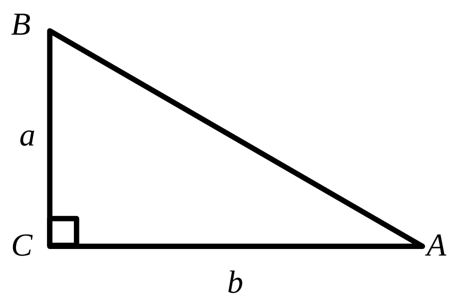
The Weekly Rigor

No. 145 "A mathematician is a machine for turning coffee into theorems." April 1, 2017

SAT Math Test Problem Children: Randomized Problem Set 1

(Part 5)

28.



Given the right triangle *ABC* above, which of the following is equal to $\frac{a}{b}$?

A) sin *A*

B) sin *B*

C) tan *A*

D) tan *B*

29. If $g(x) = 3x + 2$ and $f(x) = g(x) + 5$, what is $f(2)$?

30. In a right triangle, one angle measures $x°$, where $\sin x° = \frac{1}{2}$. What is $\cos(90° - x°)$?

ANSWERS

1. 4	7. 225	13. 0.8	19. $6+8i$	25. A
2. C	8. $\frac{3}{4}$	14. $\frac{1}{5}$	20. 4	26. 10
3. -1 or -7	9. 33	15. C	21. $\frac{20}{13}$	27. B
4. $\frac{3}{2}$	10. -3	16. $-6, 6$	22. $\frac{4}{5}$	28. C
5. $6x+6$	11. $\{6\}$	17. A	23. $-2+\sqrt{3}$, $-2-\sqrt{3}$	29. 13
6. 9	12. C	18. D	24. D	30. $\frac{1}{2}$

SELECTED SOLUTIONS

2. We are given two acute angles consisting of $a°$ and $b°$such that $\sin(a°) = \cos(b°)$. By the complementary angle relationship, $\sin(a°) = \cos(90° - a°)$. Hence, $b° = 90° - a°$, by substitution. So, $a° + b° = 90°$. Thus, since $a = 2k - 20$ and $b = 8k - 15$, it follows that $(2k - 20) + (8k - 15) = 90$. Hence, $10k - 35 = 90$, i.e., $10k = 125$. Therefore, $k = \frac{125}{10} = 12.5$, answer C.

4. The condition that the system has no solution implies that the two equations represent two distinct but parallel lines. Hence, the slopes of the lines are the same. Converting the equations into slope-intercept form, we have

$$kx - 2y = 5 \quad \Longrightarrow \quad kx - 5 = 2y \quad \Longrightarrow \quad \frac{k}{2}x - \frac{5}{2} = y \quad \Longrightarrow \quad y = \frac{k}{2}x - \frac{5}{2}$$

and

$$3x - 4y = 8 \quad \Longrightarrow \quad 3x - 8 = 4y \quad \Longrightarrow \quad \frac{3}{4}x - \frac{8}{4} = y \quad \Longrightarrow \quad y = \frac{3}{4}x - 2\,.$$

(Note that the equations have distinct *y*-intercepts.) So,

$$\frac{k}{2} = \frac{3}{4}\,.$$

Thus,

$$k = \frac{6}{4} = \frac{3}{2}$$

is the value of k that will render the system of equations to have no solution.