The Weekly Rigor

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SAT Math Test Problem Children: Randomized Problem Set 1

(Part 6)

11. This problem has three steps: 1. Substitute the given value of a into the equation; 2. Solve for *x*; 3. Test the solutions in the original equations to check for extraneous solutions. Following this procedure, we have:

$$\sqrt{x-2} = x - 4$$

$$\left(\sqrt{x-2}\right)^2 = (x-4)^2 \quad \Rightarrow \quad x - 2 = x^2 - 8x + 16 \quad \Rightarrow \quad 0 = x^2 - 9x + 18$$

$$\Rightarrow \quad 0 = (x-6)(x-3) \quad \Rightarrow \quad x = 6 \text{ and } x = 3$$

Testing the solutions, we have

$$\sqrt{6-2} \overset{?}{=} 6 - 4$$

$$\sqrt{4} \overset{\checkmark}{=} 2$$

However,

$$\sqrt{3-2} \overset{?}{=} 3 - 4$$

$$\sqrt{1} \neq -1$$

Hence, the solution set must omit 3.

18. Before using the quadratic formula, we need to arrange the terms of the equation in the standard order, in order to correctly identify the constants a, b, and c. Hence, we have

$$x^2 - \frac{k}{2}x - 2p = 0\,.$$

We will make the problem easier to solve if we eliminate the fraction in the linear term. We can accomplish this by multiplying each term by 2. So, we will have

$$2x^2 - kx - 4p = 0\,.$$

Now, using the quadratic formula, with $a = 2$, $b = -k$, and $c = -4p$, we have

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(2)(-4p)}}{2(2)} = \frac{k \pm \sqrt{k^2 + 32p}}{4} = \frac{k}{4} \pm \frac{\sqrt{k^2 + 32p}}{4}\,.$$

Therefore, the answer is option D.

21.

$$\begin{aligned}\frac{6-i}{3-2i} &= \frac{(6-i)}{(3-2i)} \cdot \frac{(3+2i)}{(3+2i)} \\ &= \frac{18 + 9i - 2i^2}{9 - 4i^2} \\ &= \frac{20 + 9i}{13} \\ &= \frac{20}{13} + \frac{9i}{13}\end{aligned}$$

Hence, a, the real part, is the number $\frac{20}{13}$.