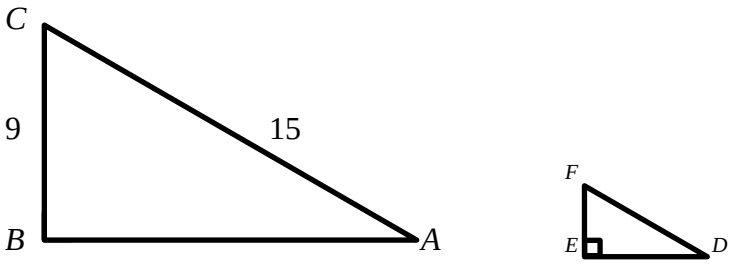
The Weekly Rigor

No. 147 "A mathematician is a machine for turning coffee into theorems." April 15, 2017

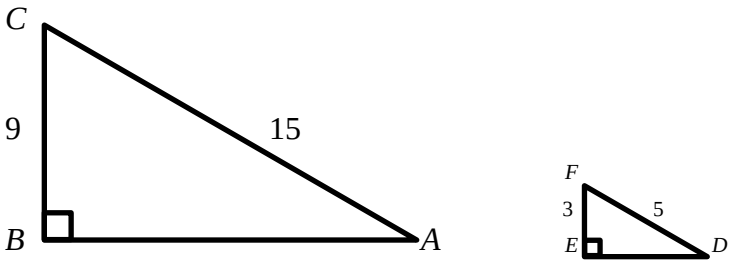
SAT Math Test Problem Children: Randomized Problem Set 1

(Part 7)

22. To solve this problem, begin by drawing representative triangles.



Since the sides of triangle *DEF* are $\frac{1}{3}$ the length of the corresponding sides of triangle *ABC*, we can fill in the lengths of *EF* and *DF*:



The value of sin *F* is equal the ratio $\frac{DE}{DF}$. Hence, we need to use the Pythagorean Theorem to find the length of *DE*. $DE^2 + 3^2 = 5^2$. So, $DE^2 = 25 - 9 = 16$. Thus, $DE = 4$. Therefore, $\sin F = \frac{4}{5}$.

24. First note that the problem states $b = c - \frac{1}{2}$. Hence, by substitution into the first equation,

$$2x + \left(c - \frac{1}{2}\right) = 4x - 6.$$

So,

$$c = 2x - 6 + \frac{1}{2} = 2x - \frac{12}{2} + \frac{1}{2} = 2x - \frac{11}{2}.$$

Thus, by substitution into the second equation,

$$2y + \left(2x - \frac{11}{2}\right) = 4y - 6.$$

Hence,

$$2x = 2y - 6 + \frac{11}{2} = 2y - \frac{12}{2} + \frac{11}{2} = 2y - \frac{1}{2}.$$

So,

$$x = y - \frac{1}{4},$$

which is option D.

25.

$$\begin{aligned}
\frac{1-3i}{6+2i} &= \frac{1-3i}{6+2i} \cdot \frac{6-2i}{6-2i} \\
&= \frac{(1-3i)}{(6+2i)} \cdot \frac{(6-2i)}{(6-2i)} \\
&= \frac{1 \cdot 6 - 1 \cdot 2i - 6 \cdot 3i + 3 \cdot 2i^2}{6 \cdot 6 - 6 \cdot 2i + 6 \cdot 2i - 2 \cdot 2i^2} \\
&= \frac{6 - 2i - 18i + 6i^2}{36 - 12i + 12i - 4i^2} \\
&= \frac{6 - 20i + 6i^2}{36 - 4i^2} \\
&= \frac{6 - 20i + 6(-1)}{36 - 4(-1)} \\
&= \frac{6 - 20i - 6}{36 + 4} \\
&= \frac{-20i}{40} \\
&= -\frac{i}{2},
\end{aligned}$$

which is option B.