

The Weekly Rigor

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“A mathematician is a machine for turning coffee into theorems.”

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Proof of a Corollary to Axiom 2 of Edmund Landau’s *Foundation of Analysis*

INTRODUCTION

In 1930, a book by Edmund Landau was published in Germany, the full title of which is *Foundations of Analysis: The Arithmetic of Whole, Rational, Irrational and Complex Numbers: A Supplement to Text-Books on the Differential and Integral Calculus*. At the beginning of this work, Landau states five Peano axioms for the set of all natural numbers. After stating the second axiom, he asserts without proof “Thus, if $x = y$, then $x' = y'$.” The purpose of this article is to fill in the proof, after stating the necessary material from his book needed to complete the proof. In the following, small italic letters will stand for natural numbers.

Definition 1: If x is given and y is given, then either x and y are the same number; this may be written

$$x = y$$

(= to be read “equals”);

or x and y are not the same number; this may be written

$$x \neq y$$

(\neq to be read “is not equal to”).

Axiom 2: For each x there exists exactly one natural number, called the successor of x , which will be denoted by x' .

Corollary to Axiom 2: If

$$x = y$$

then

$$x' = y'.$$

Proof: Suppose that

$$x = y.$$

Hence, x is the same number as y , by Definition 1. We may form x' and y' , by Axiom 2, viz., x' is a successor to x . So, y' is also a successor to x , since x and y are the same number. Therefore,

$$x' = y',$$

by Axiom 2. ■

“Only he who never plays, never loses.”

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