The Weekly Rigor

No. 160 “A mathematician is a machine for turning coffee into theorems.” July 15, 2017

The Fundamental Properties of Real Numbers

In later issues, we will make reference to the following properties of real numbers. In these properties, *a*, *b*, and *c* denote real numbers.

Axiom 1 (Closure): The sum, $a + b$ and the product, ab, are unique real numbers.

Axiom 2 (Commutation):

1. $a + b = b + a$
2. $ab = ba$

Axiom 3 (Association):

1. $a + (b + c) = (a + b) + c$
2. $a(bc) = (ab)c$

Axiom 4 (Distribution):

1. $a(b + c) = ab + ac$
2. $(a + b)c = ac + bc$

Axiom 5 (Identity Elements):

1. Additive Identity: There exists a real number 0 such that $a + 0 = 0 + a = a$ for every a.
2. Multiplicative Identity: There exists a real number 1 (different from 0) such that $a \cdot 1 = 1 \cdot a = a$ for every a.

Axiom 6 (Inverse Elements):

1. Additive Inverse: For every a there exists a real number called “the additive inverse of a,” denoted by “$-a$,” such that $a + (-a) = -a + a = 0$.
2. Multiplicative Inverse or Reciprocal: For every non-zero a there exists a real number called “the multiplicative inverse of a,” denoted by “$\frac{1}{a}$,” such that $a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$.

Fundamental Addition Property: If $a = b$, then $a + c = b + c$.

Proof: Suppose that

$$a = b.$$

But

$$a + c = a + c.$$

Therefore,

$$a + c = b + c,$$

by substitution.

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Remark: Without special comment, we shall include in this property the variation

If $a = b$, then $c + a = c + b$.

Fundamental Multiplication Property: If $a = b$, then $ac = bc$.

Proof: Suppose that

$$a = b.$$

But

$$ac = ac.$$

Therefore,

$$ac = bc,$$

by substitution.

■

Remark: Without special comment, we shall include in this property the variation

If $a = b$, then $ca = cb$.